A MANUSH characterisation of HDI

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This note is on the MANUSH axioms (signifying Monotonicity, Anonymity, Normalisation, Uniformity, Shortfall Sensitivity, and Hiatus sensitivity to level), a set of characteristics that a Human Development Index (HDI) should satisfy. It also proposes a measure $\mathcal{H}_\alpha$ that for α≥2 satisfies the MANUSH axioms. A special case of this is the additive inverse of the distance from the ideal, the displaced ideal measure, when α=2. For its axiomatic advantages, we propose to make use of the displaced ideal measure, $\mathcal{H}_2$, in the computation of HDI replacing the current geometric mean measure.

Measuring HDI: Three measures

The Human Development Index (HDI), since its inception in 1990, provides a country-specific indicator that is aggregated across three normalised unit length dimensions − health ($\hbar$, representing how long and healthy a life one lives), education (e, representing knowledge) and income (y, as a proxy for standard of living).[1] This is an important departure from income-based measures that focused on a single dimension.

HDI is an indicator aggregated across three dimensions − health, education, and income.

Prior to 2010, HDI was obtained as a linear average (or, simple arithmetic mean) of the normalised scores corresponding to the three dimensions, $\mathcal{H}_1=(\hbar+e+y)/3$. Since 2010, the three dimensions are aggregated in terms of the geometric mean, $\mathcal{H}_g=(\hbar*e*y)^{1/3}$. We propose an alternative measure of aggregation by taking the additive inverse of the Euclidean distance from the ideal,[2] $\mathcal{H}_2=1-\{[(1-\hbar)^2+(1-e)^2+(1-y)^2]/3\}^{1/2}$, referred to as the displaced ideal measure and is a special case of the $\mathcal{H}_\alpha$ measure, $\mathcal{H}_\alpha=1-\{[(1-\hbar)^\alpha+(1-e)^\alpha+(1-y)^\alpha]/3\}^{1/\alpha}$ for α=2.

The alternative measure of aggregation suggests taking the additive inverse of the Euclidean distance from the ideal and is referred to as the displaced ideal measure.

The MANUSH Axioms

We propose a set of six intuitive characteristics as axioms that is desirable for a measure of HDI. The axioms are as follows.

We propose six intuitive characteristics as axioms − Monotonicity, Anonymity, Normalisation, Uniformity, Shortfall sensitivity, and Hiatus sensitivity to level.

Monotonicity (M): A measure of HDI should be such that an increase (decrease) in attainment in any one of the three dimensions, keeping attainments in the other two dimensions constant, leads to an increase (decrease) in the value of HDI. For instance, given education and income scores, if health score is 0.1 in situation #1 and 0.2 in situation #2, then HDI(#2)>HDI(#1).

Anonymity (A): This is a symmetry condition. A measure of HDI should be invariant with respect to interchange of attainment levels across dimensions. In other words, all the six possible ways in which any three independent scores (say, 0.1. 0.4 and 0.7) can be attributed to health, education and income should have the same HDI. For instance, given income

[1] Choice of dimension-specific indicators, units of measurement of these dimension-specific indicators before their normalisation to unit length, their normalisation procedures, and weights attached to each dimension or its sub-components are important issues but beyond the scope of the current exercise.

[2] Ideal refers to a situation where all three normalised unit length dimensions take the value of unity.

score of 0.4, if health and education scores are (0.1, 0.7) in situation #1 and (0.7, 0.1) in situation #2 then HDI(#1)=HDI(#2).

Normalisation (N): A measure of HDI should lie between zero and unity such that if attainments in all the three dimensions are zero (unity) then the HDI value should be zero (unity). This imposes minimum and maximum bounds on the value of HDI. Minimum corresponds to no attainment in any of the dimensions and maximum corresponds to full attainment in all the dimensions.[3]

Uniformity (U): A measure of HDI should be such that for a given average attainment a greater deviation (or, spread) across dimensions should give a lower HDI value. For instance, given income, if health and education scores are (0.1, 0.7) in situation #1 and (0.2, 0.6) in situation #2 then HDI(#2)>HDI(#1). This axiom rewards balanced or uniform development across dimensions. The need for a balanced development in these three dimensions is also motivated by the fact that they are intrinsic, and that these ends are important means. They being important means also implies a virtuous link highlighting their instrumental relevance.[4]

For a given average attainment, a greater deviation across dimensions should give a lower HDI value.

Shortfall sensitivity (S): This is about a change (say, increment) in HDI value for a measure of HDI along its optimal path from an initial position.[5] The optimal path for health and education for different HDI measures are given in Figure 1.

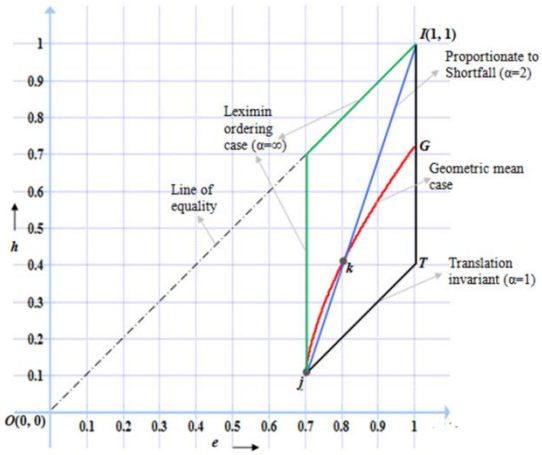

Figure 1: Optimal Paths for different measures

A measure of HDI should be such that for a given increment in the HDI value along the optimal path of the measure the increments across dimensions should be at least in proportion to the shortfalls in the worse-off dimensions.

For a given increment in the HDI value along the optimal path of the measure, the increment across dimensions should be at least in proportion to the shortfalls in the worse-off dimensions.

In other words, if $\mathcal{h}<e<\mathcal{y}$ then $\Delta\mathcal{h}\geq[(1-\mathcal{h})/(1-e)]\Delta e$ and $\Delta e\geq[(1-e)/(1-\mathcal{y})]\Delta\mathcal{y}$. For instance, in a situation where health, education and income scores are (0.1, 0.7, 0.9) indicating that shortfalls are (0.9, 0.3, 0.1), respectively, then the emphasis on health should be at least thrice the emphasis on education, while the emphasis on education should be at least thrice the emphasis on income.[6] The axiom of shortfall sensitivity follows from the notion that all dimensions of development are intrinsically important and it is desirable that all dimensions

[3] A stricter version of this axiom is as follows: a measure of HDI should lie between zero and unity such that if attainments in all the three dimensions have a common score then the HDI value should be equal to this common score, which also includes the minimum and maximum bounds of zero and unity, respectively.

[4] See Amartya Sen, *Development as Freedom*, 1999, for a discussion on instrumental relevance of ends that are also means.

[5] For any measure of HDI, an increment from an initial position can happen in multiple ways, but only the one that minimises the distance for the measure of HDI will be its optimal path.

[6] Any decision on future emphasis needs to move away from linear distances arising out of dimension-specific normalised scores to the pre normalised indicator to the real aspects that these indicators represent. However, as conveyed earlier (see, footnote 1) these are beyond the scope of the current exercise.

progress concurrently. Notwithstanding the concerns on comparisons across dimensions using a linear scale (see footnotes 1 and 6), there is some merit to articulate a case for the need to attain equal level of development across dimensions. This not only ensures greater emphasis on dimensions that are lagging behind and thereby reduces gaps across dimensions, but also makes all dimensions reach their respective ideal together.

Hiatus sensitivity to level (H): A measure of HDI should be such that the same gap (or hiatus) across dimensions should be considered worse-off as the average attainment increases. A measure of HDI satisfies hiatus sensitivity to level when for the same gap the deviation of HDI value from its corresponding average attainment increases with increase in average attainment. For instance, if the scores of health, education and income and corresponding HDI are (0.1, 0.4 0.7; 0.4) in situation #1 and (0.2, 0.5, 0.8; 0.5) in situation #2 then HDI(#1), in comparison to HDI(#2), should have a lower deviation from its average attainment.

The same gap across dimensions should be considered worse-off as the average attainment increases.

This is in line with development with equity across dimensions. It imposes that the same gap across dimensions would be considered worse-off as average attainment increases. For instance, in a society, where health and education have lagged behind the income-based standard of living, it is not desirable for these gaps to persist with further development. Thus, for any development involving more than one dimension, higher overall attainment must simultaneously lead to a reduction in achievement gap across dimensions.

Comparing Measures through MANUSH

The arithmetic mean measure, $\mathcal{H}_1$, satisfies the first three axioms or the MAN axioms. The geometric mean measure, $\mathcal{H}_g$, satisfies the first four axioms or the MANU axioms (with an exception condition for M when any of the dimensions has a normalised value of zero to begin with). The α-distance measure, $\mathcal{H}_\alpha$, satisfies all the six axioms or the MANUSH axioms for α≥2. Incidentally, an anagram of MANUSH is HUMANS.

A special case of $\mathcal{H}_\alpha$ measure is when α=2, the additive inverse of the Euclidean distance from the ideal, which is referred to as the displaced ideal measure, $\mathcal{H}_2$. We propose a MANUSH or HUMANS characterisation of HDI and call for the use of the displaced ideal measure in the computation of HDI replacing the current geometric mean.

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