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A MANUSH or HUMANS characterisation of the Human Development Index

Srijit Mishra
Hippu Salk Kristle Nathan

Nabakrushna Choudhury Centre for Development Studies, Bhubaneswar
(an ICSSR institute in collaboration with Government of Odisha)

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Srijit Mishra,[2] Hippu Salk Kristle Nathan[3]

Abstract

Proposing a set of axioms MANUSH (Monotonicity, Anonymity, Normalisation, Uniformity, Shortfall sensitivity, Hiatus sensitivity to level) this paper evaluates three aggregation methods of computing Human Development Index (HDI). The old measure of HDI, which is a linear average of the three dimensions, satisfies monotonicity, anonymity, and normalisation (or MAN) axioms. The current geometric mean approach additionally satisfies the axiom of uniformity, which penalises unbalanced development across dimensions. We propose $\mathcal{H}_\alpha$ measure, which for $\alpha \geq 2$ also satisfies axioms of shortfall sensitivity (emphases on the worse-off to better-off dimensions should be at least in proportion to their shortfalls) and hiatus sensitivity to level (higher overall attainment must simultaneously lead to a reduction in gap across dimensions). Special cases of $\mathcal{H}_\alpha$ are the linear average ($\alpha=1$), the displaced ideal ($\alpha=2$), and the leximin ordering ($\alpha \to \infty$) methods. For its axiomatic advantages, we propose to make use of the displaced ideal ($\alpha=2$) method in the computation of HDI replacing the current geometric mean.

Key words: Displaced ideal, Hiatus sensitivity, MANUSH, Shortfall sensitivity, Uniform development

JEL codes: D63, I31, O15

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[2] Director, Nabakrushna Choudhury Centre for Development Studies (NCDS), Bhubaneswar, Odisha, E-mail: srijit.ncds@gov.in.

[3] Assistant Professor, National Institute of Advanced Studies (NIAS), IISc Campus, Bangalore-560012, E-mail: happyhippu@gmail.com.

1. Introduction

In the human development paradigm the emphasis is on human beings as *ends* in themselves and not so much as *means* of development.[4] Further, the ends are in multiple dimensions. Mahbubul Haq, the founder of *Human Development Reports* (HDRs),[5] considers one-dimensionality as the most serious drawback of the income-based measures. This led to the birth of the Human Development Index (HDI), see Haq (1995, chapter 4). The measurement of HDI has evolved over time and has contributed significantly to policy discourse.[6]

The calculation of HDI involves three dimensions—health, education, and the ability to achieve a decent standard of living, represented by income. The attainments of each country in these three dimensions are normalised,[7] and then aggregated to get the composite HDI. Prior to 2010, linear averaging across attainments in three dimensions was used as an aggregation method to obtain HDI.[8] This method of aggregation, which implies perfect substitutability across dimensions has been criticised in the literature for being inappropriate (Desai, 1991; Hopkins, 1991; Palazzi and Lauri, 1998; Sagar and Najam, 1998; Raworth and Stewart, 2003, Herrero *et al*., 2010a). Perfect substitutability "amounts to saying, for instance, that no matter how bad the health state is, it can be compensated with further education or additional income, *at a constant rate*" (Herrero *et al*., 2010a: 4). According to Sagar and Najam (1998: 251), such "a reductionist view of human development is completely

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[4] For discussions on this, see Streeten *et al.* (1981), Sen (1989, 1997, 1999, and 2000), Desai (1991), Streeten (1994), and Haq (1995), among others.
[5] The human development report is being published annually since 1990 and serves as a cornerstone in terms of philosophy as well as an approach of the United Nations Development Programme (UNDP).
[6] For discussions on birth, evolution, measurement, and critique of HDI and its policy discourse, see Anand and Sen (1993, 1995, 1997, 2000, and 2003), Haq (1995); Lüchters and Menkhoff (1996); Dutta *et al*. (1997); Hicks (1997); Noorbakhsh (1998); Sen (2000); Panigrahi and Sivaramakrishna (2002); Fukuda-Parr *et al*. (2003); Jahan (2003); Raworth and Stewart (2003); Ranis *et al*. (2006); Grimm *et al.* (2008); Alkire and Foster (2010); Nathan and Mishra (2010); United Nations Development Program (2010); Klugman *et al*. (2011); Wolff *et al*. (2011); Harttgen and Klasen (2012); Ravallion (2012); and Permanyer (2013), among others.
[7] The normalisation used: value=(actual-minimum)/(maximum-minimum).
[8] In this paper, attainment refers to the normalised value of the indicators representing the dimensions of HDI.

contrary to the UNDP's own definition." It contradicts the notion of human development, where, as Sen (1999) suggests, each dimension is intrinsic.

Acknowledging the above-mentioned limitation, the 20th anniversary edition of human development report (UNDP, 2010) changed the aggregation method to geometric mean.[9] In this paper, we propose an alternative aggregation method, which is the additive inverse of the distance from the ideal.[10] Following Zeleny (1982), we refer to this as the displaced ideal method.[11]

As a first step, this paper evaluates the above-mentioned three aggregation methods using a set of axioms, namely, *Monotonicity*, *Anonymity*, *Normalisation*, *Uniformity*, *Shortfall sensitivity* and *Hiatus sensitivity to level*, which we refer as MANUSH.[12] There have been previous works by Chakravarty (2003), Foster *et al.* (2005), Seth (2009), and Zambrano (2014) towards axiomatic characterisation of HDI. Some of these papers do refer to the axioms of monotonicity, anonymity, and normalisation, whereas the remaining three axioms proposed in the current exercise are new. We don't claim MANUSH to be an exhaustive list of desirable properties of a human development measure. We also acknowledge that the focus of the paper is on aggregation of attainments in different dimensions, and not on the aggregation of attainments of individuals or subgroups. Further, the paper considers the choice of the three dimensions and how they are measured, scaled, weighed, or normalised as given.[13]

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[9] The trade-off across dimensions in this method, as indicated by Ravallion (2010) and Chakravarty (2011) is troubling. Zambrano (2016) has also responded to this concern.

[10] The ideal corresponds to the maximum values for all the three dimensions as posited by UNDP for the calculation of the human development index. In this sense, ideal indicates complete attainment. We use distance in the Euclidean sense unless otherwise specified.

[11] Chakravarty and Majumder (2008) suggest the use of shortfalls in targets while evaluating the progress of Millennium Development Goals.

[12] MANUSH (or *manush*) means human beings in some of the South Asians languages such as Assamese, Bengali, Marathi, and Sanskrit, among others. Incidentally, the term has HUMANS as its anagram. In this sense, the paper proposes the axiom of HUMANS for the human development index.

[13] There are other deserving alternative choices in dimensions, measurement, weights, and normalisation, and these may have an ambiguity in capability space, quite similar to real income space as discussed in Sen (1979).

The pre-2010 linear average method satisfies monotonicity, anonymity, and normalisation. The current method of geometric mean satisfies these three axioms, albeit an exception condition that it fails monotonicity if attainment in any dimension remains at zero. It also satisfies the uniformity axiom by penalising unbalanced development across dimensions. Our proposed displaced ideal method satisfies the above four axioms. Additionally, it satisfies axioms of shortfall sensitivity (emphases on the worse-off to better-off dimensions should be at least in proportion to their shortfalls) and hiatus sensitivity to level (higher overall attainment must simultaneously lead to a reduction in gap across dimensions).

In the second step, we propose $\mathcal{H}_\alpha$ measure where, α is 'aversion to inequality' across dimensions. Special cases of this measure turn out to be the linear average method (α=1), the displaced ideal method (α=2), and the leximin ordering ($\alpha \to \infty$). We also show that $\mathcal{H}_\alpha$ measure for $\alpha \geq 2$ satisfy MANUSH axioms.

Though the focus of the current paper has been on aggregation of attainments across dimensions, the method can be applied to aggregation across subgroups. There have been attempts to make the human development index measure sensitive to address inequality across individuals or subgroups within each dimension (Hicks, 1997; Chatterjee, 2005), or, to address inequality both between and within dimensions (Chakravarty, 2003; Foster *et al*., 2005; Seth, 2009).[14]

The rest of the paper is organised as follows. Section 2 introduces the notations and definitions. The three aggregation methods are discussed in Section 3 and the MANUSH axioms are elaborated in Section 4. On the basis of these axioms, the three methods of aggregation are compared in Section 5. Section 6 proposes $\mathcal{H}_\alpha$ measure and evaluates this as

[14] In a measure of deprivation that involves multiple dimensions, the priority of computing individual deprivation has been indicated by Dutta *et al*. (2003). Our suggested alternative is a method of aggregation that can also be applied to individual data.

also some other measures of HDI against the MANUSH axioms. Concluding remarks are given in Section 7.

2. Notation and definitions

The human development index is an aggregation measure $\mathcal{H}_i$; $i = \alpha, \beta, g, C, F$. It is computed from attainments in multiple dimensions, x_j; $j = 1, \ldots, n$. Here, n refers to the number of dimensions.[15] For ease of analysis, in a three dimensional HDI space, we also use $x_j = \hbar, e, y$ to denote attainments in health, education, and income, respectively. The mean and a standard deviation of attainments across dimensions are denoted by μ and σ, respectively. Attainments in each dimension is normalised such that, $0 \leq x_j \leq 1 \forall j$. A human development measure is a function $f\colon [1,0]^n \to \mathcal{R}$, where $\mathcal{R}$ denotes the set of real numbers.

$$\mathcal{H}_\alpha = 1 - \left[\left\{\sum\left(1 - x_j\right)^\alpha\right\}/n\right]^{(1/\alpha)}; \alpha = 1, \ldots, \infty, \quad (1)$$

is a measure proposed in this paper.[16] It follows that for $\alpha=1$, $\mathcal{H}_1$ is the linear average of attainments in the three dimensions, $\sum x_j/3$, and for $\alpha=2$, $\mathcal{H}_2$ is the displaced ideal method. $\mathcal{H}_\beta$ is an alternative measure.

$\mathcal{H}_g$ is the geometric mean of the attainments in three dimensions, $\Pi x_j^{1/3}$. $\mathcal{H}_\mathcal{C}$ is a measure proposed by Chakravarty (2003), $(\sum x_j^\delta)/3$; $\delta \in (0,1]$, and at its lower bound when $\delta=1$, $\mathcal{H}_\mathcal{C}=\mathcal{H}_1$. Based on Atkinson's index, Foster *et al*. (2005) proposed a measure, $\mathcal{H}_\mathcal{F}=(\sum x_j^{(1-\varepsilon)}/3)^{1/(1-\varepsilon)}$ for $\varepsilon \geq 0$, $\varepsilon \neq 1$. For $\varepsilon=1$, $\mathcal{H}_\mathcal{F}=\mathcal{H}_g$. Note that $\mathcal{H}_\mathcal{F}=\mathcal{H}_1$ at $\varepsilon=0$, $\mathcal{H}_\mathcal{F}$ takes the

[15] The HDI calculation involves three dimensions. However, for generalisation purpose, we have considered n dimensions.

[16] This measure takes the form of Minkowski distance function. Use of Minkowski distance function in the context of human development is not new. Prior to 2010, the *Human Development Reports* used Minkowski distance function across different dimensions of deprivations to calculate Human Poverty Indices (HPI-1 and HPI-2), see Anand and Sen (1997) and Pillai (2004). Subramanian (2006) has also used the Minkowski distance function to the Foster *et al*. (1984) poverty measure.

form of a harmonic mean at ε=2 and, $\mathcal{H}_{\mathcal{C}}=\mathcal{H}_{\mathcal{F}}^{(1-\varepsilon)}$ as ε=1-δ when $\varepsilon\in[0,1)$.

For comparison across situations, k, we use $\mathcal{H}_{ik}$, x_{jk}, μ_k, σ_k; $k=a,b$. We define gap, $\gamma_{jk}=\mu_k-x_{jk}$, to be the difference between the mean and the attainment in dimension j. For subgroup, s, population share and $\mathcal{H}_i$ are denoted by η_s and $\mathcal{H}_{is}$, respectively. In the appendices, $\mathcal{D}$ is the distance between two situations, and z_1 to z_8 are constants.

We have the following formal definitions:

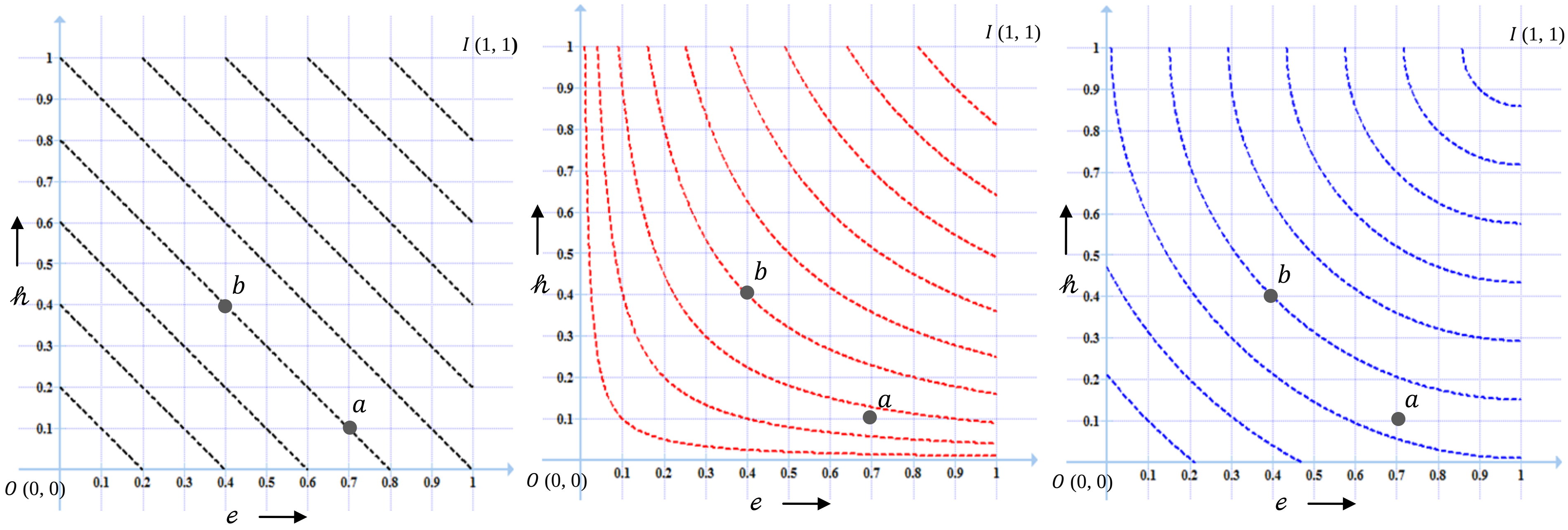
Origin and *Ideal*: When attainments in all the dimensions of $\mathcal{H}_i$ are at their minima, $x_j=0\forall j$, or maxima, $x_j=1\forall j$, then we refer to the situations as the origin, O, or the ideal, I, respectively.

Uniform development and *deviation from uniform development*: When all the dimensions of $\mathcal{H}_i$ have equal attainments, $x_j=\mu\forall j$, then we refer to the situations as uniform development, ${}_u\mathcal{H}_i$. The deviation of $\mathcal{H}_i$ from ${}_u\mathcal{H}_i$, is denoted by $\mathcal{V}_i$ such that; $\mathcal{V}_i={}_u\mathcal{H}_i-\mathcal{H}_i$.

Optimal path: From an initial position, for a given increase in the value of $\mathcal{H}_i$, there can be many possible paths. From all these possibilities, the one that is identified with the minimum distance is the optimal path. In other words, for a given distance, a progress along the optimal path maximises the increase in the value of $\mathcal{H}_i$.

3. The three methods of aggregation

Prior to 2010, HDRs used $\mathcal{H}_1$. An implicit assumption of $\mathcal{H}_1$ is that attainments across the three dimensions are perfectly substitutable. This means that an increase in attainment in one dimension can be substituted by an equal decrease in another dimension. From 2010, HDRs used $\mathcal{H}_g$. It does not allow perfect substitutability and gives higher importance to increment in the dimension having lower attainment. In other words, it penalises unbalanced development (Gidwitz *et al*., 2010; Herrero *et al*., 2010b; Kovacevic and Aguña, 2010).



(a) Iso-$\mathcal{H}_1$ (b) Iso-$\mathcal{H}_g$ (c) Iso-$\mathcal{H}_2$

Figure 1: Iso-$\boldsymbol{\mathcal{H}_i}$ lines in two two-dimensional space

We propose a measure, $\mathcal{H}_2$, based on the notion that a better situation should have less distance from the ideal, and refer to this as the displaced ideal method (Zeleny, 1982). This measure is the additive inverse of the normalised distance from the ideal, $1-[\sqrt{\{\sum(1-x_j)^2\}}/\sqrt{3}]$. $\mathcal{H}_2$, like $\mathcal{H}_g$, penalises unbalanced development.

The distinction among the three methods of aggregation, $\mathcal{H}_1$, $\mathcal{H}_g$, and $\mathcal{H}_2$, can be represented in terms of iso-$\mathcal{H}_i$ surfaces. Without loss of generalisation, in a two-dimensional space (h,e), they would correspond to 45^0 inclined (or backward hatched) lines (Figure 1a), rectangular hyperbola lines (Figure 1b), and concentric quarter circles (Figure 1c), respectively. In these figures, it is seen that a (h=0.1, e=0.7) is equal to b (h=0.4, e=0.4) in terms of $\mathcal{H}_1$, which is on account of perfect substitutability. However, a is less than b in terms of $\mathcal{H}_g$ and $\mathcal{H}_2$. For further distinction among the three methods, we propose to compare them through a set of axioms.

4. The MANUSH axioms

This section presents a number of intuitive properties that $\mathcal{H}_i$ should satisfy. They are as follows.

Axiom M - *Monotonicity*:
$\mathcal{H}_{ia}<\mathcal{H}_{ib}$ if x_{jb} is obtained from x_{ja} by an increment.

An increase (decrease) in the attainment in any of the three dimensions (h,e,y) while the attainments of the other two dimensions remain constant will lead to an increase (decrease) in the value of $\mathcal{H}_i$. For instance, with e and y remaining constant, if in one situation h=0.1 while in another situation h=0.2, then between the two situations, the former will have a lower $\mathcal{H}_i$ value. This is same as the monotonicity axiom used by Chakravarty (2003). Foster *et al*. (2005) and Seth (2009) use this to imply that $\mathcal{H}_i$ will increase if the attainment of any one person in any single dimension increases.

Axiom A – Anonymity:

$\mathcal{H}_{ia}$=$\mathcal{H}_{ib}$ if x_{jb} is obtained from x_{ja} by a permutation.[17]

This is a symmetry condition. If there are two situations where the attainments of the dimensions get interchanged then they will give the same $\mathcal{H}_i$ value. For instance, with y remaining constant, if in one situation h=0.1 and e=0.7 while in another situation h=0.7 and e=0.1, then in both the situations $\mathcal{H}_i$ will have same value. This axiom is the same as 'symmetry in dimension axiom' of Foster *et al*. (2005) and Seth (2009) and is used in the same spirit as 'aggregation symmetry axiom' of Zambrano (2014).

Axiom N – *Normalisation*:
$\mathcal{H} \in [0,1]$; $\mathcal{H}_i$=0 at O and $\mathcal{H}_i$=1 at I.

If attainments in all the three dimensions are zero (unity) then value of $\mathcal{H}_i$ should be zero (unity). This imposes minimum and maximum bounds on the value of $\mathcal{H}_i$. Minimum corresponds to no attainment in any of the dimensions and maximum corresponds to full attainment in all the dimensions. In other words, if h=0, e=0, and y=0, then $\mathcal{H}_i$=0, and if h=1, e=1, and y=1, then $\mathcal{H}_i$=1. The way we use the axiom of normalisation is similar to the inequality literature. However, in the human development literature it means that if all dimensions have a common value then $\mathcal{H}_i$ will be equal to this common value (Chakravarty, 2003; Foster *et al.* 2005; Seth 2009; Zambrano 2014). It is implicit in this that $\mathcal{H}_i$ will be zero and unity representing origin and ideal, respectively.

Axiom U – *Uniformity*:
$\mathcal{H}_{ia}$<$\mathcal{H}_{ib}$ if x_{jb} is obtained from x_{ja} such that μ_a=μ_b and σ_a>σ_b. For a given average attainment across dimensions (μ_k), a greater deviation (σ_k) should give a lower $\mathcal{H}_i$ value. For instance, with y remaining constant, if in one situation h=0.1 and e=0.7 while in another

[17] In the computation of HDI in the human development reports, the three dimensions are given equal weights. However if they were to be given different weights, permutation assumes interchanging of values together with weights.

situation $\mathcal{h}$=0.2 and e=0.6, then between the two situations, the former will have a lower $\mathcal{H}_i$ value. This axiom rewards balanced or uniform development across dimensions. The need for a balanced development of these intrinsic dimensions (or capabilities, restricted to long and healthy life, knowledge, and standard of living in the $\mathcal{H}_i$ measure) is also motivated by the fact that these ends are important means, or, in other words, their instrumental relevance and the virtuous link between them also favour a balanced development across dimensions (Sen, 1999).

Axiom S – *Shortfall sensitivity*:

An increase in the value of $\mathcal{H}_i$, $(\mathcal{H}_{ib}-\mathcal{H}_{ia})>0$, along the optimal path should be such that the increment in attainments across dimensions ($x_{jb}-x_{ja}\geq 0 \forall j$) satisfy

$(x_{1b}-x_{1a})/(1-x_{1a})\geq(x_{2b}-x_{2a})/(1-x_{2a})\geq(x_{3b}-x_{3a})/(1-x_{3a})$; $x_{1k}<x_{2k}<x_{3k}$, $k=a,b$ for all distinct $x_1,x_2,x_3 \in \{\mathcal{h},e,y\}$ (2)

This axiom means that an increase in the value of $\mathcal{H}_i$ along the optimal path should be such that the increment across the worse-off to better-off dimensions should be at least in proportion to their shortfalls. This also follows from the notion that all dimensions of development are intrinsically important and it is desirable to attain equal level of development across dimensions. To address this, the future emphases on the worse-off to better-off dimensions should be at least in proportion to their shortfalls. For instance, in a situation where $\mathcal{h}$=0.1, e=0.7, and y=0.9 (indicating that shortfalls are 0.9, 0.3, and 0.1, respectively) then the emphasis on health should be at least thrice the emphasis on education, while the emphasis on education should be at least thrice the emphasis on income. This not only ensures greater emphasis on dimensions that are lagging behind and thereby reduces gaps across dimensions, but also makes all dimensions reach their respective ideal together.[18]

[18] In the context of distribution of resources among individuals, shortfall sensitivity will be a stricter condition than that of the equity axiom of Hammond (1976) or priority axiom of Moreno-Ternero and Roemer (2006),

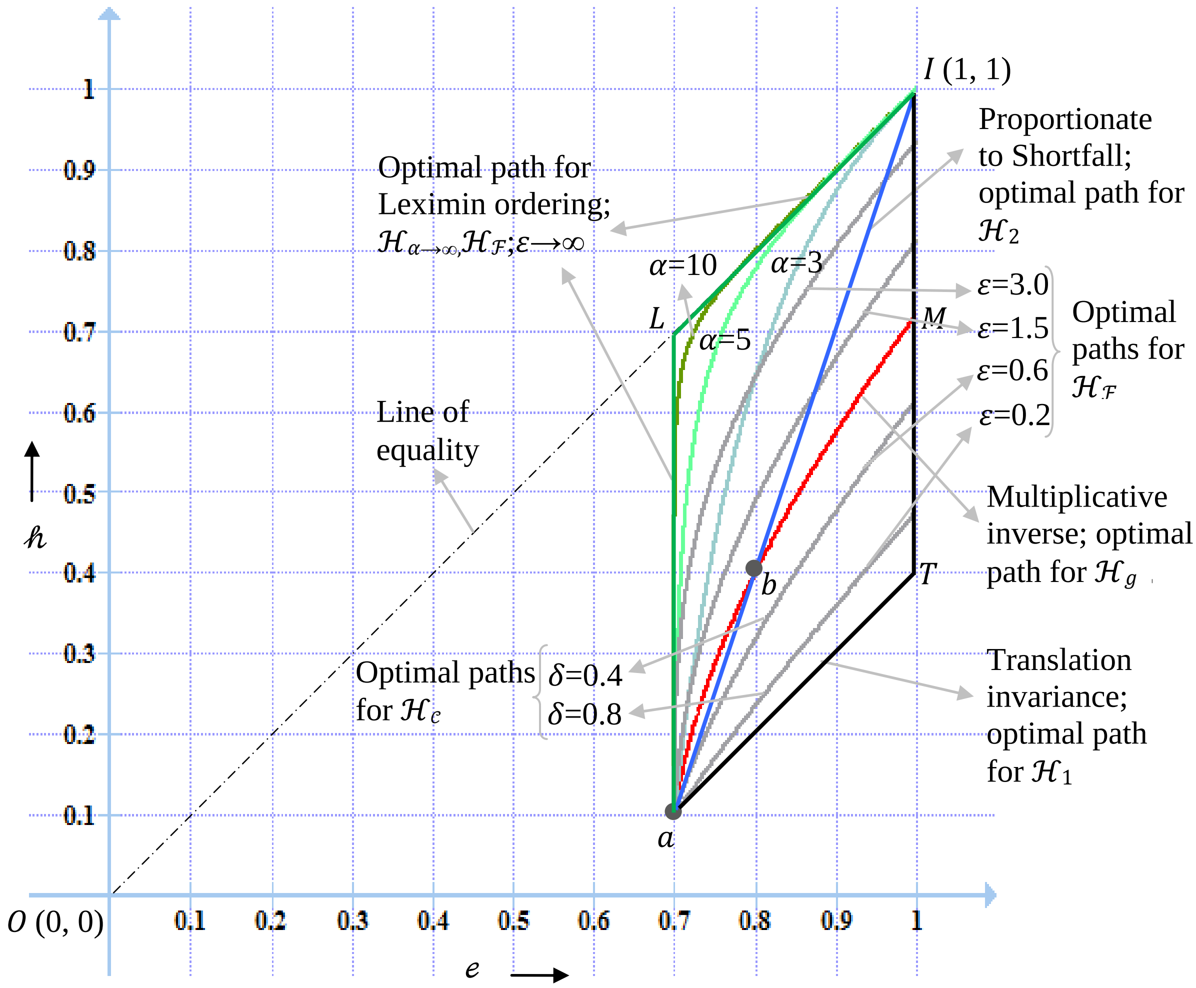


Figure 2. *Shortfall sensitivity and optimal paths for different measures*

An exacting situation of this equity consideration is to give the entire emphasis to the most neglected dimension till it becomes equal to the dimension that is ordered just above it. And then the entire emphasis will be shared equally across both these dimensions till they reach to the dimension that is ordered above them, and then all the three dimensions will get equal emphasis. We refer to this as the leximin ordering. The lines aL and LI together (in Figure 2) indicate the leximin ordering. Such leximin ordering is identified with Rawls'

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both of which draw on the weak equity axiom of Sen (1973, p.18-19) to indicate that a greater emphasis is to be given to the worse-off.

justice in the space of distributions of resources among individuals,[19] while we use the notion of leximin ordering in the space of development across dimensions.

Axiom H – *Hiatus sensitivity to level*:
$({}_u\mathcal{H}_{ia}$-$\mathcal{H}_{ia}) < ({}_u\mathcal{H}_{ib}$-$\mathcal{H}_{ib})$ if $\gamma_{ja}=\gamma_{jb}\forall j$ and $\mu_a<\mu_b$.

This means that the same gap (or hiatus) across dimensions should be considered worse off as the attainment increases. A way to evaluate the sensitivity of $\mathcal{H}_i$ to hiatus is to examine $\mathcal{V}_i=({}_u\mathcal{H}_{ia}$-$\mathcal{H}_{ia})$ i.e., its deviation from the uniform development case where all the dimensions have equal attainments such that $x_j=\mu\forall j$. $\mathcal{H}_i$ satisfies hiatus sensitivity to level when $\mathcal{V}_i$ decreases with increase in μ. For instance, if in one situation h=0.1, e=0.4, and y=0.7 while in another situation h=0.2, e=0.5, and y=0.8, then between the two situations, the former will have a lower $\mathcal{V}_i$ value.

This is in line with development with equity across dimensions. It imposes that the same gap across dimensions would be considered worse off as average attainment increases. For instance, in a society, where health and education have lagged behind the income-based standard of living, it is not desirable for these gaps to persist with further development. Thus, for any development constituting more than one dimension, higher overall attainment must simultaneously lead to a reduction in gap across dimensions. This is similar to the level sensitivity axiom in the context of group differential measures (Mishra and Subramanian, 2006; Mishra, 2008; Nathan and Mishra, 2013).[20]

The above set of axioms, namely, monotonocity, anonymity, normalisation, uniformity, shortfall sensitivity, and hiatus sensitivity to level are collectively referred to as MANUSH, or its anagram HUMANS.

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[19] This lexicographic extension of the difference principle by Rawls (1971, 2001) is the only generalised social welfare function in the context of interpersonal ordering, which satisfies the Arrow conditions, the equity axiom, and Suppes' grading principle (Hammond, 1976).

[20] In the space of distribution of income among individuals, level sensitivity refers to society’s greater concern for inequity with increase in prosperity, as suggested by Sen (1973).

4. Axiomatic Comparison

The three aggregation methods of computing HDI, viz., $\mathcal{H}_1$, $\mathcal{H}_g$, and $\mathcal{H}_2$, satisfy the axioms of monotonicity (with an exception condition for $\mathcal{H}_g$), anonymity, and normalisation. Further, $\mathcal{H}_g$ and $\mathcal{H}_2$ methods satisfy the axiom of uniformity. The axioms of shortfall sensitivity and hiatus sensitivity to level are satisfied by $\mathcal{H}_2$ method alone. Let us elaborate.

Monotonicity: This axiom is satisfied for all the three methods with an exception condition for $\mathcal{H}_g$. For two situations a and b if the attainment in one dimension is higher for one, with attainments in the other dimensions being the same, say, $\hbar_a<\hbar_b$, while $e_a=e_b$, and $y_a=y_b$, then $\mathcal{H}_{1a}<\mathcal{H}_{1b}$, $\mathcal{H}_{ga}<\mathcal{H}_{gb}$, and $\mathcal{H}_{2a}<\mathcal{H}_{2b}$. This fails for $\mathcal{H}_g$ when e or y=0.

Anonymity: The three methods of aggregation satisfy anonymity as they are symmetric in $\hbar$, e, and y.

Normalisation: In all the three methods, the values of $\mathcal{H}_i$ are bounded by the minimum, $\mathcal{H}_1=\mathcal{H}_g=\mathcal{H}_2=0$ at the origin, O; and the maximum, $\mathcal{H}_1=\mathcal{H}_g=\mathcal{H}_2=1$ at the ideal, I. Hence, they satisfy normalisation.

Uniformity: Both $\mathcal{H}_g$ and $\mathcal{H}_2$ methods satisfy this, while $\mathcal{H}_1$ fails. For two situations a and b, if $\mu_a=\mu_b$ and $\sigma_a>\sigma_b$ then $\mathcal{H}_{ga}<\mathcal{H}_{gb}$ and $\mathcal{H}_{2a}<\mathcal{H}_{2b}$, but $\mathcal{H}_{1a}=\mathcal{H}_{1b}$.

This axiom along with anonymity implies that for a given mean, $\mathcal{H}_g$ and $\mathcal{H}_2$ are maximised when all the three dimensions have equal values ($\hbar=e=y$). Further, this axiom and the property of perfect substitutability cannot be satisfied together, as given in Proposition 1.

Proposition 1
$\mathcal{H}_i$ cannot satisfy perfect substitutability and uniformity simultaneously.

Proof: $\mathcal{H}_i$ satisfying perfect substitutability means that for a given μ the value of $\mathcal{H}_i$ is constant. On the contrary, $\mathcal{H}_i$ satisfying uniformity means that for a given μ the value of $\mathcal{H}_i$ decreases (increases) as σ increases (decreases).

As we know, $\mathcal{H}_1$ satisfies perfect substitutability. In fact, as discussed earlier, this is "one of the most serious criticisms of the linear aggregation formula" (UNDP, 2010: 216).

Shortfall sensitivity: $\mathcal{H}_i$ is shortfall sensitive if increments in worse-off to better-off dimensions along the optimal path are at least in proportion to their shortfalls. As indicated earlier, the optimal path requires minimising the distance between the initial and any subsequent position for a given increase in the value of $\mathcal{H}_i$. In other words, for any $\mathcal{H}_i$, a progress along the optimal path maximises the increase in the value of $\mathcal{H}_i$ for a given distance between the initial position and any subsequent position.

For $\mathcal{H}_1$, an optimal path will be perpendicular to the iso-$\mathcal{H}_1$ line. A progress along the optimal path will imply same increment in all dimensions. For the position a in Figure 2 the optimal path will be aT, which is identified with the translation invariance case. This way, under $\mathcal{H}_1$, the emphases across dimensions are equal and they are independent of their current values. It does not impose greater emphases on the dimensions that have been hitherto neglected. Thus, $\mathcal{H}_1$ fails in shortfall sensitivity.

For $\mathcal{H}_g$, an optimal path will be such that the progress along it gives the emphases across dimensions that are in proportion to the multiplicative inverse of attainments (for proof, see Appendix 1). For the position a in Figure 2, the optimal path will be aM. The bM segment of the path is outside the area aIL, which indicates that beyond b (at b the optimal path for $\mathcal{H}_g$ intersects the proportionate to shortfall line aI) it imposes emphases less than in proportion to shortfalls on the dimensions that are relatively lower. Hence, $\mathcal{H}_g$ fails in shortfall sensitivity.

For $\mathcal{H}_2$, an optimal path will be the line joining the initial position and the ideal (for proof, see Appendix 2). For the position a in Figure 2, such a path is aI. Here the emphases across dimensions are in proportion to their shortfalls throughout the path. Thus, $\mathcal{H}_2$ satisfies shortfall sensitivity weakly.

Hiatus sensitivity to level: $\mathcal{H}_1$ fails to satisfy this axiom, as there is no deviation of $\mathcal{H}_1$ from its uniform development case. For a given gap, the deviation of $\mathcal{H}_i$ from its uniform development case is a decreasing function of mean for $\mathcal{H}_g$ while it is an increasing function of mean for $\mathcal{H}_2$ (for proof, see Appendix 3). This implies that $\mathcal{H}_g$ fails in hiatus sensitivity to level, whereas $\mathcal{H}_2$ satisfies.

From the above discussion the following results emerge. $\mathcal{H}_1$ satisfies the axioms of MAN (monotonicity, anonymity, and normalisation). In addition to these, $\mathcal{H}_g$ satisfies the axiom of uniformity, while it also has an exception condition for monotonicity where it fails. $\mathcal{H}_2$ satisfies the aforementioned four axioms and additionally the axioms of shortfall sensitivity (weakly) and hiatus sensitivity to level. Based on this, we state Proposition 2.

Proposition 2

$\mathcal{H}_2$ satisfies the MANUSH axioms– monotonicity, anonymity, normalisation, uniformity, shortfall sensitivity (weakly) and hiatus sensitivity to level.

Thus, $\mathcal{H}_2$ has some axiomatic advantages over $\mathcal{H}_1$ and $\mathcal{H}_g$. Nevertheless one may mention that an advantage of $\mathcal{H}_1$ is that of subgroup decomposability. However, it should not be seen as an advantage as it implies that the subgroups having lower value of $\mathcal{H}_i$ values can be perfectly substitutable by subgroups having higher $\mathcal{H}_i$ values. Additionally, our proposed method can also be made subgroup decomposable by considering the s^{th} subgroup's share of contribution as $\eta_s \mathcal{H}_{is} / \sum \eta_s \mathcal{H}_{is}$.

Similarly, one must also mention that an advantage of $\mathcal{H}_g$ is that the ranking of countries are independent to changes in the maximum value for each dimension, which is used for normalising the attainments in each dimension. However this advantage is redundant if one follows the practice of fixing the maximum, in a normative sense, as a goalpost. The practise of computing $\mathcal{H}_g$ in HDRs used an open-ended maximum in 2010, but it got reverted back to a fixed maximum in 2014.

5. The $\mathcal{H}_\alpha$ Measure

In the $\mathcal{H}_\alpha$ measure, the linear average ($\mathcal{H}_1$) and the displaced ideal ($\mathcal{H}_2$), as indicated earlier, are special cases. Another special case is $\mathcal{H}_{\alpha\rightarrow\infty}$, where the value of the measure reduces to the attainment of dimension having the lowest value. This corresponds to a situation where the iso-$\mathcal{H}_\alpha$ lines can be depicted through right-angled lines. Thus, as α increases from unity to infinity we move from a measure that allows for perfect substitutability to one that allows no substitution across dimensions (Figure 3).[21]

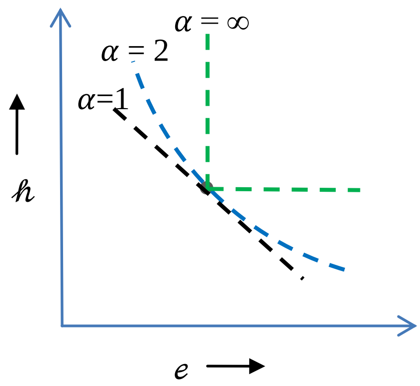

Figure 3. $\mathcal{H}_\alpha$ *Measure*

$\mathcal{H}_\alpha$ measure satisfies the MANUSH axioms for $\alpha \geq 2$. We state that in Lemma 1.

[21] The similarity of $\mathcal{H}_\alpha$ measures with constant elasticity of substitution (CES) functions is obvious (Rao, 2011).

Lemma 1

For every α, such that $\alpha \geq 2$, the human development measure $\mathcal{H}_\alpha$ satisfies the MANUSH axioms.

Proof: The proof for the six axioms is as follows. $\mathcal{H}_\alpha$ satisfies monotonicity, $d\mathcal{H}_\alpha/dx_j>0\forall j$. It satisfies anonymity, as $\mathcal{H}_\alpha$ remains the same if x_js are interchanged. It satisfies normalisation, as $\mathcal{H}_\alpha \in [0,1]$. It satisfies uniformity, $d\mathcal{H}_\alpha/dx_j>0$ and $d^2\mathcal{H}_\alpha/(x_j)^2<0\forall j$. Shortfall sensitivity is not satisfied when $\alpha<2$, it is weakly satisfied for $\alpha=2$, and satisfied for $\alpha>2$ (Appendix 2). $\mathcal{H}_\alpha$ satisfies hiatus sensitivity to level for $\alpha \geq 2$ (Appendix 3).

The optimal paths of $\mathcal{H}_\alpha$ for $\alpha=1,2$, and ∞ are given in Figure 2 indicating cases of translation invariance, proportionate to shortfall, and leximin ordering, respectively. For values of $\alpha \in (2,\infty)$ the paths will be within the area aIL and concave to the line segment aI; some sample optimal paths for $\alpha=3$, 5, and 10 are also given in Figure 2. The choice of α (or substitution across dimensions) is intertwined with the degree of shortfall sensitivity.[22] A value of $\alpha \geq 2$ gives us a multitude of options between proportionate to shortfall and leximin ordering ($\alpha \to \infty$) that satisfy shortfall sensitivity and in that sense make all such possibilities reasonably plural, and hence, Rawlsian in spirit.[23]

The existing measures such as $\mathcal{H}_\mathcal{C}$ (Chakravarthy, 2003) and $\mathcal{H}_\mathcal{F}$ (Foster *et al.*, 2005), which is also similar to Seth (2009) with regard to aggregation across dimensions, satisfy monotonicity, anonymity, normalisation, and uniformity, but they fail to satisfy shortfall sensitivity and hiatus sensitivity to level.

The optimal paths for $\mathcal{H}_\mathcal{C}$ and $\mathcal{H}_\mathcal{F}$ are given for different values of δ and ε,

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[22] For $\mathcal{H}_\alpha$, the minimum bound of α is restricted to 1, i.e., the condition where the optimal path gives equal emphases to all the dimensions in future progress. For $\alpha<1$, the optimal paths are such that the emphasis given to the neglected dimension is less than the emphasis given to favored dimension. Also, note that for $\alpha<1$, we will have iso-HDI lines that are concave to the origin.

[23] Sen (2000, 2009), while critiquing other aspects, surmise that a major contribution of Rawls (1971) is the invoking of reasonable pluralism. This has larger implications and is different from leximin ordering identified with his difference principle. For an application of Rawls' reasonable pluralism on conflict resolution, see Mishra (2011). In the current context, leximin ordering is a special case of the Rawlsian spirit; an exacting situation from the multitude of possibilities that satisfy shortfall sensitivity.

respectively, in Figure 2.[24] Both do not satisfy shortfall sensitivity as they either fall below the proportionate to shortfalls line aI or intersect it and thereby reaching the maximum relatively earlier for the dimension that is doing relatively better (except for the limiting case in $\mathcal{H}_{\mathcal{F}}$ when $\varepsilon\rightarrow\infty$; that coincides with $\mathcal{H}_{\alpha}$; $\alpha\rightarrow\infty$). For $\mathcal{H}_{\mathcal{C}}$, the optimal path coincides with that of $\mathcal{H}_g$ in a limiting sense when $\delta\rightarrow 0$ and as δ increases it goes rightward and when δ=1 it coincides with the translation invariance case identified with linear average, i.e., $\mathcal{H}_1$. For $\mathcal{H}_{\mathcal{F}}$, the optimal paths are same as that of $\mathcal{H}_{\mathcal{C}}$ when $\varepsilon<1$ as $\varepsilon=1-\delta$. It coincides with that of $\mathcal{H}_1$ at ε=0 and that of $\mathcal{H}_g$ at ε=1. As ε increases the relative emphasis on the neglected dimension increases, but it fails in shortfall sensitivity for all finite values of ε (as the optimal paths intersect aI line like that of $\mathcal{H}_g$, Figure 2). For illustration, in Figure 2 we have given optimal paths for $\mathcal{H}_{\mathcal{F}}$ when ε=0.2, 0.6, 1.5, and 3 and $\mathcal{H}_{\mathcal{C}}$ when δ=0.4 and 0.8.

Both $\mathcal{H}_{\mathcal{C}}$ and $\mathcal{H}_{\mathcal{F}}$ do not satisfy hiatus sensitivity. For a given gap, the deviations of $\mathcal{H}_{\mathcal{C}}$ and $\mathcal{H}_{\mathcal{F}}$ from their corresponding uniform development case will be lower for a greater mean.[25]

While the focus of the current paper has been across dimensions, we have a remark on composite measure across subgroups. In Figure 2, if the two axes refer to the attainments of two subgroups in any specific indicator then the composite index based on MANUSH axioms can account for inequality across the two subgroups. In such a scenario, the shortfall sensitivity axiom would mean giving emphases on the worse-off to better-off subgroups at least in proportion to their shortfalls. And, the hiatus sensitivity axiom means that with an increase in overall attainment the gap between subgroups should reduce.

[24] The formulae for the optimal paths of $\mathcal{H}_{\mathcal{C}}$ and $\mathcal{H}_{\mathcal{F}}$ in a two-dimensional situation of $\hbar<e$ are $d\hbar/de=(e/\hbar)^{(1-\delta)}$; δ=(0,1] and $d\hbar/de=(e/\hbar)^{\varepsilon}$; $\varepsilon\geq 0$, $\varepsilon\neq 1$, respectively. The two formulae coincide in the domain $\varepsilon<1$ as $\varepsilon=1-\delta$.

[25] In fact, Chakravarthy (2003: 104) also points out that $\mathcal{H}_{\mathcal{C}}$ "will attach greater weight to achievement differences at lower level of attainment." Thus, confirming our observation that it fails hiatus sensitivity to level.

Now, suppose we have an alternative measure of human development index, $\mathcal{H}_\beta$, that also satisfies the MANUSH axioms then it means the following. $\mathcal{H}_\beta$ is an increasing function for each dimension (monotonicity). The function associated with $\mathcal{H}_\beta$ will be symmetric across dimensions (anonymity). There will be bounds to $\mathcal{H}_\beta$ such that it lies between zero and unity (normalisation). The function increases at a decreasing rate: iso-$\mathcal{H}_\beta$ curves should be convex to the origin (uniformity). $\mathcal{H}_\beta$ satisfies equation (2), (shortfall sensitivity). And, for a given gap, a fall in $\mathcal{H}_\beta$ from its corresponding uniform development case will be greater for a greater mean (hiatus sensitivity to level).

By using equation (2), which defines shortfall sensitivity, we obtain an optimal path for $\mathcal{H}_\beta$ and this happens to be equivalent to the one that we obtained by using $\mathcal{H}_\alpha$ then we can conclude that $\mathcal{H}_\beta$ has a one to one correspondence with $\mathcal{H}_\alpha$. With this, we propose Lemma 2 and a theorem.

Lemma 2

If a human development measure f satisfies MANUSH axioms then it must be

$\mathcal{H}_\alpha$for some α, such that $\alpha \geq 2$.

Proof: Appendix 4.

Theorem 1

A human development measure f satisfies the MANUSH axioms if and only if there

exists α, $\alpha \geq 2$, such that f is $\mathcal{H}_\alpha$

Proof: Lemmas 1 and 2.

One can state that sensitivity to shortfalls increases as α increases such that as $\alpha \to \infty$ it can be identified with a leximin ordering. In targeting and policy intervention for specific situations, α may be appropriately increased.

6. Conclusions

This exercise evaluated three methods of aggregation across dimensions for measuring human development index through a set of intuitive axiomatic properties. The linear averaging method satisfied the axioms of monotonicty, anonymity, and normalisation (or MAN axioms). The geometric mean method, in addition to these three axioms (excluding monotonicity when one of the dimensions continues to have a value of zero), also satisfied the axiom of uniformity. The proposed displaced ideal method (additive inverse of the distance from the ideal) satisfied the above-mentioned four axioms as also the axioms of shortfall sensitivity (emphases on the worse-off to better-off dimensions should be at least in proportion to their shortfalls) and hiatus sensitivity to level (higher overall attainment must simultaneously lead to a reduction in gap across dimensions). We refer to this set of axioms as MANUSH axioms. The word MANUSH has additional significance: it means human in many South Asian Languages and is also an anagram of HUMANS.

We also propose $\mathcal{H}_\alpha$ measure where α=1, α=2, and $\alpha\to\infty$ turned out to be the linear averaging method, the displaced ideal method, and the leximin ordering, respectively. Further, for the $\mathcal{H}_\alpha$ measure $\alpha\geq2$, the MANUSH axioms are satisfied. Thus, giving us multitude of options that are reasonably plural and thereby Rawlsian in spirit.

While acknowledging the advantage of a multitude of options, a simple beginning is to make use of the displaced ideal method (α=2) in the calculation of the human development index. It not only has axiomatic advantages over the linear average and geometric mean methods but also in terms of shortfall sensitivity, its optimal path being proportionate to shortfall, lies between the translation invariance case (α=1) associated with the linear average method that gives no premium to historical antecedents and the exacting case of leximin ordering ($\alpha=\infty$) where the entire emphasis is on the worse-off dimension. In this backdrop, we propose to make use of the displaced ideal ($\mathcal{H}_2$) method in the computation of HDI replacing the current geometric mean.

Our articulation in the current exercise was to do with the measurement of human development index. However, it would also be relevant for other indices computed through multidimensional aggregation. For instance, it could be used in the construct of Global Hunger Index (GHI), and Social Institutions and Gender Index (SIGI), among others. If the dimensions have different weights, the measure can be appropriately weight adjusted. Further, in our proposed $\mathcal{H}_\alpha$ measure, one can consider the dimensions as subgroups. The discussion of translation invariance and leximin ordering are important constructs in the context of inequality across groups and the discussion of hiatus sensitivity to level have also been borrowed from literature on group differential.

Appendices

Appendix 1

From the initial position a and a variable incremental position b we obtain two constants,

$$z_1=(\mathcal{H}_{gb}-\mathcal{H}_{ga})=\Pi x_{jb}^{1/3}-\Pi x_{ja}^{1/3}, \quad (A1.1)$$

$$z_2=\Pi x_{jb}=(z_1+\Pi x_{ja}^{1/3})^3. \quad (A1.2)$$

Note that $x_{jk}=h_k, e_k, y_k$ and $k=a,b$. We denote $\mathcal{D}$, as the distance from a to b and minimise,

$$\mathcal{D}^2=\sum(x_{jb}-x_{ja})^2 \quad (A1.3)$$

Substituting y_b from (A1.2) to (A1.3) and applying the minimisation conditions $(\mathcal{D}^2)/dh_b$ and $(\mathcal{D}^2)/de_b$ and similarly proceeding after substituting for e_b one gets,

$$h_b-h_a=e(e_b-e_a)=y_b(y_b-y_a) \quad (A1.4)$$

The equation for optimal path can be determined by considering infinitesimally small increment. Integrating the first part of (A1.4) we get,

$$\int h\,dh=e\,de \Rightarrow h^2=e^2+z_3 \quad (A1.5)$$

where z_3 is constant. From the initial position we get that $z_3=h_a^2-e_a^2$. It follows that,

$$h^2=e^2+h_a^2-e_a^2 \quad (A1.6)$$

Similarly, from the second part of (A1.4)

$$e^2=y^2+e_a^2-y_a^2 \quad (A1.7)$$

From (A1.6) and (A1.7) we get

$$y^2=h^2+y_a^2-h_a^2 \quad (A1.8)$$

Appendix 2

From the initial position a and a variable incremental position b we obtain two constants,

$$z_4=1-[\{\sum(1-x_{jb})^{\alpha}\}/3]^{(1/\alpha)}-1+[\{\sum(1-x_{ja})^{\alpha}\}/3]^{(1/\alpha)} \quad (A2.1)$$

$$z_5=[\{\sum(1-x_{ja})^{\alpha}\}/3]^{(1/\alpha)} \quad (A2.2)$$

Note that $x_{jk}=h_k, e_k, y_k$ and $k=a,b$. Expressing y_b in terms of h_b and e_b we get,

$$y_b=1-\{3(z_5-z_4)-(1-h_b)-(1-e_b)\}^{(1/\alpha)} \quad (A2.3)$$

As in Appendix 1, we minimise,

$$\mathcal{D}^2=\sum(x_{jb}-x_{ja})^2 \quad (A2.4)$$

Substituting and applying the minimisation conditions, as in Appendix 1, one gets,

$$(h_b-h_a)/\{(1-h_b)^{(\alpha-1)}\}=(e_b-e_a)/\{(1-e_b)^{(\alpha-1)}\}=(y_b-y_a)/\{(1-y_b)^{(\alpha-1)}\} \quad (A2.5)$$

The equation for optimal path can be determined by considering infinitesimally small increment and then integrating. From the first part of (A2.5) we get,

$$d\hbar/de=\{(1-\hbar)/(1-e)\}^{(\alpha-1)}. \quad \text{(A2.6)}$$

For α=1, corresponding to $\mathcal{H}_1$, $d\hbar/de$=1. Applying this to the initial position one gets the optimal path, which coincides with the translation invariance case (Figure 2). Extending to three dimensions we get,

$$d\hbar/de=d\hbar/dy=de/dy=1. \quad \text{(A2.7)}$$

For α=2, corresponding to $\mathcal{H}_2$, rearranging and integrating (2.6) gives,

$$\int d\hbar/(1-\hbar)=\int de/(1-e)\Rightarrow \ln(1-\hbar)=\ln(1-e)+z_6\Rightarrow 1-\hbar=z_7(1-e) \quad \text{(A2.8)}$$

where z_6 and z_7 are constants. As a will be on the optimal path, $z_7=(1-\hbar_a)/(1-e_a)$. Thus,

$$\hbar=\{(1-\hbar_a)/(1-e_a)\}e+\{(\hbar_a-e_a)/(1-e_a)\}. \quad \text{(A2.9)}$$

This shows the proportion to shortfall case (Figure 2). Extending to three dimensions we get,

$$d\hbar/(1-\hbar)=de/(1-e)=dy/(1-y). \quad \text{(A2.10)}$$

For α>2, rearranging and integrating (2.6) gives

$$\int d\hbar/(1-\hbar)^{(\alpha-1)}=\int de/(1-e)^{(\alpha-1)}\Rightarrow[\{(1-\hbar)^{(2-\alpha)}\}/(2-\alpha)]=[\{(1-\hbar)^{(2-\alpha)}\}/(2-\alpha)]+z_8 \quad \text{(A2.11)}$$

where z_8 is constant. As a will be on the optimal path, $z_8=[\{(1-\hbar_a)^{(2-\alpha)}\}-\{(1-e_a)^{(2-\alpha)}\}]/(2-\alpha)$. Substituting z_8 in (A2.11) and simplifying,

$$\hbar=1-\{(1-e)^{(2-\alpha)}+(1-\hbar_a)^{(2-\alpha)}-(1-e_a)^{(2-\alpha)}\}^{1/(2-\alpha)}. \quad \text{(A2.12)}$$

Extending to three dimensions we get,

$$d\hbar/(1-\hbar)^{(\alpha-1)}=de/(1-e)^{(\alpha-1)}=dy/(1-y)^{(\alpha-1)} \quad \text{(A2.13)}$$

Appendix 3

For two situations with same gap $\gamma_j=\gamma_{ja}=\gamma_{jb}\forall j$. Let $\mathcal{V}_g$ be the deviation of $\mathcal{H}_g$ from the uniform development situation, μ, such that,

$$\mathcal{V}_g=\mu-\Pi x_j^{1/3}=\mu-\Pi(\mu-\gamma_j)^{1/3} \quad \text{(A3.1)}$$

Note that $x_j=\hbar,e,y$. Differentiating $\mathcal{V}_g$ with μ we get,

$$d\mathcal{V}_g/d\mu=1-(1/3)\{\Pi(\mu-\gamma_j)^{-2/3}\}[\{(\mu-\hbar)(\mu-e)\}+\{(\mu-\hbar)(\mu-y)\}+\{(\mu-e)(\mu-y)\}] \quad \text{(A3.2)}$$

Simplifying (A3.2) gives us,

$$1-(\Pi x_j^{1/3})(1/3)[\{(\hbar e)+(\hbar y)+(ey)\}/\Pi x_j]$$

$$=1-\{(\text{geometric mean})/(\text{harmonic mean})\} \quad \text{(A3.3)}$$

Note that (geometric mean)≥(harmonic mean). It follows that $d\mathcal{V}_g/d\mu\leq 0$. Equality holds when there is no deviation from uniform development. Thus, $\mathcal{V}_g$ is a decreasing function of μ.

Now, let $\mathcal{V}_\alpha$ be the deviation of $\mathcal{H}_\alpha$ from the uniform development situation, μ, such that,

$$\mathcal{V}_\alpha=\mu-1-[\{\sum(1-x_j)\}/3]^{(1/\alpha)} \quad \text{(A3.4)}$$

Substituting x_j with μ-$\gamma_j$and differentiating $\mathcal{V}_\alpha$ with μ and simplifying gives,

$$d\mathcal{V}_\alpha/d\mu=1-[\{\sum(1-x_j)^{-1}\}/\{3^{(1/\alpha)}\{\sum(1-x_j)^\alpha\}^{(\alpha-1)/\alpha}\}] \quad \text{(A3.5)}$$

In (A3.4) $\{\sum(1-x_j)^{-1}\}\leq\{3^{(1/\alpha)}\{\sum(1-x_j)^\alpha\}^{(\alpha-1)/\alpha}\}$, it follows that $d\mathcal{V}_\alpha/d\mu\geq 0$. Equality holds when there is no deviation from uniform development. Thus, $\mathcal{V}_g$ is a decreasing function of μ and $\mathcal{V}_\alpha$ is an increasing function of μ.

Appendix 4

For $\mathcal{H}_\beta$; $h<e<y$ to satisfy (2) for h and e implies

$$dh/de\geq(1-h)/(1-e) \quad \text{(A4.1)}$$

Note that (1-h)>(1-e) and $\mathcal{H}_\beta$ is symmetric across dimensions (anonymity axiom), rewriting (A4.1) gives us,

$$dh/de=\{(1-h)/(1-e)\}^\beta;\ \beta\geq 1 \quad \text{(A4.2)}$$

See the similarity of the condition for optimal path of (A4.2) for $\mathcal{H}_\beta$ and that of (A2.6) for $\mathcal{H}_\alpha$. There is a one-to-one correspondence when $\beta=\alpha$-1.

References

Alkire, S., and J. Foster. 2010. "Designing the Inequality-Adjusted Human Development Index (HDI)." OPHI Working Paper 37, Oxford Poverty & Human Development Initiative, Oxford.

Anand, S., and A. K. Sen. 1993. "Human Development Index: Methodology and Measurement." Human Development Report Office Occasional Paper 12, New York: .UNDP,

Anand, S., and A. K. Sen. 1995. "Gender Inequality in Human Development: Theories and Measurement." Human Development Report Office Occasional Paper 19,New York:UNDP.

Anand, S., and A. K. Sen. 1997. "Concepts of Human Development and Poverty: A Multidimensional Perspective." Human Development Papers, New York: United Nations Publications.

Anand, S., and A. K. Sen. 2000."The Income Component of the Human Development Index." *Journal of Human Development* 1(1): 83–106.

Anand, S., and A. K. Sen. 2003. "Human Development Index: Methodology and Measurement." In *Readings in Human Development*, edited by S. Fukuda-Parr and A. K. Shiva Kumar, 138–151. New York: Oxford University Press.

Chakravarty, S. R. 2003. "A Generalized Human Development Index." *Review of Development Economics* 7(1): 99-114.

Chakravarty, S. R. 2011. "A reconsideration of the tradeoffs in the new human development index" *Journal of Economic Inequality* 9: 471-474.

Chakravarty, S. R., and A. Majumder. 2008. "Millennium Development Goals: Measuring Progress towards their Achievement." *Journal of Human Development* 9(1): 109–129.

Chatterjee, S. K. 2005. "Measurement of Human Development: an alternative approach." *Journal of Human Development* 6(1): 31–44.

Desai, M. 1991. "Human Developments – Concepts and measurement." *European Economic Review*35(2–3): 350–357.

Dutta, B., M. Panda, and W. Wadhwa. 1997. "Human Development in India." In *Measurement of Inequality and Poverty*, edited by S. Subramanian, 329–357. New Delhi: Oxford University Press.

Dutta, I., P. Pattanaik, and Y. Xu. 2003. "On Measuring Deprivation and the Standard of Living in a Multidimensional Framework on the Basis of Aggregate Data," *Economica* 70(2) 197-221.

Fakuda-Parr, S., K. Raworth, and A. K. Shiva Kumar.2003 "Using the HDI for Policy Analysis." In *Readings in Human Development,* edited by S. Fukuda-Parr and A. K. Shiva Kumar, 177–187. New York: OxfordUniversity Press.

Foster, J. E., J. Greer, and E. Thorbecke. 1984. "A Class of Decomposable Poverty Measures." *Econometrica* 52(3): 761–766.

Foster, J. E., L. F. Calva, and M. Székely 2005. "Measuring the Distribution of Human Development: methodology and an application to Mexico." *Journal of Human Development* 6(1):5–25.

Gidwitz, Z., M. P. Heger, J. Pineda, and F. Rodríguez. 2010. 'Understanding Performance in Human Development: A Cross-National Study.' *Human Development Research Paper* 2010/42.

Grimm, H., K. Harttgen, S. Klasen, and M. Misselhorn. 2008. "A Human Development Index by Income Groups." *World Development* 36(12): 2527–2546.

Hammond, P. J. (1976), Equity, Arrow's conditions and Rawls' difference principle. *Econometrica*, 44, 793–804.

Haq, M. 1995. *Reflections of Human Development*. New York: Oxford University Press.

Harttgen, K. and S. Klasen. 2012. "A Household-Based Human Development Index." *World Development* 40 (5): 878–899.

Herrero, C., R. Martínez, and A. Villar.2010a. "Multidimensional Social Evaluation-An Application to the Measurement of Human Development." *Review of Income and* Wealth 56(3): 483–497.

Herrero, C., R. Martínez, and A. Villar. 2010b. "Improving the Measurement of Human Development." *Human Development Research Paper* 2010/12.

Hicks, D. 1997. "The Inequality-Adjusted Human Development Index: A Constructive Proposal." *World Development* 25(8): 1283–1298.

Hopkins, M. 1991. "Human development revisited: A new UNDP report." *World Development* 19(10): 1469–1473.

Jahan, S. 2003. "Evolution of the Human Development Index." In *Readings in Human Development,* edited by S. Fukuda-Parr and A. K. Shiva Kumar, 152–163. New York: Oxford University Press.

Klugman, J., F. R. Rodriguez, and H.-J.Choi. 2011. "The HDI 2010: New Controversies, Old Critiques." *Human Development Research Paper* 2011/01.

Kovacevic, M., and C. G. Aguña. 2010. "Uncertainty and Sensitivity Analysis of the Human Development Index." *Human Development Research Paper* 2010/47.

Lüchters, G., and L. Menkhoff. 1996. "Human Development as Statistical Artefact." *World Development* 24(8): 1385–1392.

Mishra, S. 2008. "On Measuring Group Differentials Displayed by Socio-economic Indicators: An Extension." *Applied Economics Letters* 15(12): 935–938.

Mishra, S. 2011."Conflict Resolution through Mutuality: Lessons from Bayesian Updating." *Journal of Quantitative Economics* New Series 9(1): 41-52.

Mishra, U.S., and S. Subramanian. 2006. "On Measuring Group Differentials Displayed by Socio-economic Indicators." *Applied Economics Letters* 13(8): 519–521.

Moreno-Ternero, J.D., and J. E. Roemer. 2006 "Impartiality, Priority, and Solidarity in the Theory of Justice." *Econometrica* 74(5): 1419-1427.

Nathan, H. S. K., and S. Mishra. 2010 "Progress in Human Development – Are we on the Right Path?" *International Journal of Economic Policy in Emerging Economies*, 3(3): 199–221.

Nathan, H. S. K., and S. Mishra. 2017 "Group Differential for Attainment and Failure Indicators", *Journal of International Development* 29 (4): 520-532.

Noorbakhsh, F. 1998. "A Modified Human Development Index." *World Development* 26(3): 517–528.

Palazzi, P., and A. Lauri. 1998. "The Human Development Index: Suggested Corrections." *Banca Nazionale del Lavoro Quarterly Review* 51(205): 193–221.

Panigrahi, R., and S. Sivramkrishna S. 2002. "An Adjusted Human Development Index: Robust Country Rankings with Respect to the Choice of Fixed Maximum and Minimum Indicator Values." *Journal of Human Development* 3(2): 301–311.

Permanyer, I. 2013. "Using Census Data to Explore the Spatial Distribution of Human Development." *World Development* 46: 1-13.

Pillai, N. V. 2004. "CES, Generalised Mean and Human Poverty Index: Exploring Some Links," Working Paper No. 360. Thiruvananthapuram: Centre for Development Studies.

Ranis, G., F. Stewart, and E. Samman. 2006. "Human Development: Beyond the Human Development Index." *Journal of Human Development* 7(3): 323–358.

Rao, T.V.S.R. 2011. “Contemporary Relevance and Ongoing Controversies Related to the CES Production Function.” *Journal of Quantitative Economics* 9(2): 36-57.

Ravallion, M. 2012. “Troubling Tradeoffs in the Human Development Index.” *Journal of Development Studies* 99(2): 201-209.

Raworth, K., and D. Stewart. 2003. “Critiques of the Human Development Index: A Review.” In *Readings in Human Development,* edited by S. Fukuda-Parr and A. K. Shiva Kumar, 164–176. New York: Oxford University Press.

Rawls, J. B. 1971. *A Theory of Justice*, Cambridge, Mass: Harvard University Press.

Rawls, J. B. 2001. *Justice as Fairness: A Restatement*, Cambridge, Mass: Harvard University Press.

Sagar A. D., and A. Najam. 1998. “The Human Development Index: A Critical Review.” *Ecological Economics* 25(3): 249–264.

Sen, A. 1973. *On Economic Inequality*, Oxford: Clarendon Press.

Sen, A. 1979. “ The Welfare Basis of Real Income Comparisons: A Survey.” *Journal of Economic Literature* 17(1): 1-45.

Sen, A 1989. “Development as Capability Expansion.” *Journal of Development Planning* 19: 41–58.

Sen, A 1997. “Human Capital and Human Capability.” *World Development* 25(12): 1959–1961.

Sen, A 1999. *Development as Freedom*. Oxford: Oxford University Press.

Sen, A 2000. “A Decade of Human Development.” *Journal of Human Development* 1(1): 17–23.

Sen, A 2009. *The Idea of Justice*. London: Penguin.

Seth, S. 2009. “Inequality, Interactions, and Human Development.” *Journal of Human Development and Capabilities* 10(3): 375-396.

Streeten, P., J. S. Burki, M. U. Haq, N. Hicks, and F. Stewart. 1981. *First Things First: Meeting Basic Human Needs in Developing Countries*. New York: Oxford University Press.

Streeten, P. 1994 “Human development: means and ends.” *The American Economic Review* (Papers and Proceedings) 84(2): 232–237.

Subramanian, S. 2006. “A Re-scaled Version of the Foster-Greer-Thorbecke Poverty Indices based on an Association with the Minkowski Distance Function.” In *Rights, Deprivation, and Disparity: Essays in Concepts and Measurement*. Collected Essays Series, New Delhi: Oxford University Press.

United Nations Development Program. 2010. *Human Development Report2010*, *The Real Wealth of Nations: Pathways to Human Development*. New York: Oxford University Press.

Wolff, H., H. Chong and M. Auffhammer. 2011. "Classification, Detection and Consequences of Data Error: Evidence from the Human Development Index." *The Economic Journal* 121 (553), 843–870.

Zambrano, E. 2014. "An axiomatization of the human development index." *Social Choice and Welfare* 42 (4): 853-872. DOI: 10.1007/s00355-013-0756-9.

Zambrano, E. 2016. "The 'Troubling Tradeoffs' Paradox and a Resoluiton" *Review of Income and Wealth* 63 (3): 520-541. DOI: 10.1111/roiw.12235.

Zeleny, M. A. 1982. *Multiple Criteria Decision Making*. New York: McGraw-Hill Book Company.