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Bertrand Competition with One-sided Cost Uncertainty

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Abstract

This paper examines Bertrand duopoly with homogeneous products and one-sided cost uncertainty. One-sided cost uncertainty has broader relevance in many important market settings in economics, especially in the areas of market entry and cost-reducing R&D investments. We show that all undominated equilibria induce a unique equilibrium outcome in terms of firms' expected profits. We apply our results to examine cost-reducing R&D by an entrant firm which enters a monopolistic market. Productive investments can result in a low marginal cost of production, but it may simultaneously induce a harsh price competition. In such a situation, in order to reduce the resulting intense competition, the entrant would intentionally choose an inefficient level of investment as a part of its equilibrium strategy that would induce some technological uncertainty.

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1. Introduction

The difference in manufacturing technologies among firms can be a source of competitive advantages and disadvantages in any market. Specifically, in market situations where price is the strategic variable (*a la* Bertrand), such known difference in technologies have resulted in the equilibrium behavior of firms to deviate considerably from the standard marginal cost pricing outcome. Several prior studies of the standard Bertrand model with homogeneous products and asymmetric constant marginal costs, e.g., Blume (2003) [1], provide us with a class of equilibria with undominated strategies where the lowest-cost firm wins the market and earns a positive profit.

In many occasions, however, competing firms may not have complete information about each other's technology. Particularly, long presence of any business in a market reveals numerous clues to interested potential competitors about the cost function of the incumbent. On the other hand, a potential entrant to such a market may possess private information about its own cost function not immediately discoverable by its competitors. Hence, in such a case, an entrant may have full information about the incumbent's cost while the incumbent may possess incomplete information about the former's cost. As far as the authors are aware, no studies have focused on the equilibrium characterization of Bertrand competition with this class of information asymmetry, even in the most canonical setting with homogeneous products.

As a starting point for developing this branch of research, we investigate equilibrium behavior in a Bertrand duopoly with homogeneous products and one-sided cost uncertainty.[1] Specifically, we consider a situation in which one firm (hereafter, firm I) privately observes its constant marginal cost, whereas the other (uninformative firm U)'s cost is publicly known.

In this paper, we obtain a unique equilibrium outcome (Theorem 4.1) by focusing on the undominated equilibria where both firms set their prices no less than their marginal costs. In these equilibria, firm U's expected profit is determined by the highest profit that firm U is guaranteed to get when firm I set its price at its marginal cost. Furthermore, we show that the outcome of our proposed equilibria converges to the conventional equilibrium outcome of the low-cost firm serving the entire market at a price equal to the rival's marginal cost as cost uncertainty is reduced.

Although we present a stylized model in this paper, various extensions of this model are possible to suit different market situations. In Section 5, we present a major application of our model where an entrant makes a cost-reducing R&D investment decision to enter a monopolistic market. Our main findings indicate that the entrant firm (firm I in our model), whose cost function is not known to the market, can obtain an information rent due to its informational advantage of having private information. Pertinent to this, we present an interesting theoretical exercise where the entrant may manipulate its R&D investment level before it enters the market.

[1] An earlier version of this paper was included in Patra's PhD thesis (Patra (2016) [18]).

To this end, we consider the model where firm I endogenously induces one-sided cost uncertainty by conducting a cost-reducing R&D investment before it enters the market. We show that choosing an inefficient investment level would induce sufficient cost uncertainty that weakens price competition, and hence, increase firm I's expected profit. This mechanism works even when firm I's marginal cost never takes a value lower than the marginal cost of firm U no matter how much firm I invests. Because of the presence of technological disadvantage for firm I in this situation, if firm U can make an accurate estimate of the realized value of firm I's cost, firm I cannot enjoy a profit in the market even if it succeeds in achieving cost reduction due to its R&D. However, our results show that firm I can recover some profit by inducing cost uncertainty and building an informational advantage. Surprisingly, we observe that even R&D inducing a deterioration of technology can be profitable for firm I. This result holds as long as the prospective profit generated by the R&D that induces an appropriate degree of cost uncertainty is higher than the cost of such R&D.

To our knowledge, our model is the first to study Bertrand duopoly with homogeneous goods and one-sided cost uncertainty. Assuming constant cost technology, related literature to our work extends in two directions: cost asymmetry and cost uncertainty. Blume (2003) [1] investigates the standard Bertrand duopoly given asymmetric marginal costs without assuming the discrete strategy space and without resorting to ε-equilibria. This paper characterizes the undominated equilibria that achieve the conventional outcome of the low-cost firm charging a price equal to the high-cost firm's marginal cost and serving the entire market.[2] By contrast, in our model, both firms can obtain positive expected profits under the undominated equilibrium.

Continuing with the standard Bertrand model, Hansen (1988) [7], Patra (2015) [17], Routledge (2010) [19], and Spulber (1995) [22] show that if firms are uncertain about their rivals' costs, the outcome such as prices above marginal costs and positive expected profits is achievable as an equilibrium outcome. These papers assume symmetry of cost uncertainty among firms. In contrast, in our model we assume asymmetric cost uncertainty among firms, particularly, one-sided cost uncertainty.

Bertrand competition under asymmetric cost information can be viewed as a first-price sealed-bid auction in which the quantity demanded is endogenous. In our model, firms face asymmetric cost uncertainty. In the literature of asymmetric auctions, asymmetric valuation of bidders is paralleled with asymmetric cost uncertainty, which makes such auctions closely related to our study. Many prior studies (Lebrun (1996) [11], Lebrun (1999) [12], Maskin (2000) [13], Maskin (2003) [14], Jackson and Swinkels (2005) [9]) show the existence of the equilibrium in first-price sealed-bid auctions under different assumptions.[3] However, no previous research provides the full characterization of equilibria

[2] Kartik (2011) [10] and De Nijs (2012) [16] strengthen this result by showing that such an equilibrium outcome will be achieved in every equilibrium where every firm plays undominated strategies.

[3] Milgrom and Weber (1982) [15] and Engelbrecht-Wiggans et al. (1983) [6] investigate an asymmetric first-price common-value auction and show that a bidder with no private informa-

in the classical Bertrand model (the first-price sealed-bid auction) that satisfies all of the following assumptions: (i) there is one-sided type uncertainty, (ii) the winner is selected at random in the case of a tie, (iii) the type space is discrete, and the action space is continuous, and (iv) the quantity of the demand is endogenous. In the present paper, we characterize the Bayesian Nash equilibria and show the uniqueness of the undominated equilibrium.

Our application in section 5 is closely related in concept to "strategic cost-reducing R&D" and "strategic entry" in oligopoly theory similar to Brander and Spencer (1983) [2] and Dixit (1980) [5].[4] The literature emphasizes the effect of irreversible investment on market competition largely as a commitment device. In this paper, we demonstrate a new strategic aspect of "commitment effect" induced from irreversible investment. We show that using such investment strategically to introduce technological (cost) uncertainty weakens market competition and increases firms' expected profits, even when it can result in a deterioration of technology.

The paper proceeds as follows. Section 2 demonstrates our main findings in the context of motivating examples. Section 3 outlines setup. Section 4 derives main results. In section 5, we apply our results to examine the cost-reducing R&D by an entrant firm which enters a monopolistic market. Finally, section 6 offers some concluding remarks.

2. Example

We begin by demonstrating our main findings in the context of specific examples.

2.1. Price competition with one-sided cost uncertainty

Consider a situation where a risk-neutral buyer procures a single object from two risk-neutral suppliers, firm U and firm I. The buyer's budget is $p^{\max}$ at most and firm U's production cost is c_U. All of the information above are common knowledge, whereas firm I's production cost is its private information.

We assume that firm I's production cost c_I can take three discrete values, $c_I \in \{1, 2, 3\}$, each with equal probability. Firm U's cost c_U is a value strictly between 1 and 2: $1 < c_U < 2$. We also suppose that the buyer is able to afford any marginal cost pricing by firm I, i.e., $3 < p^{\max}$. Suppliers simultaneously set their prices and the buyer procures the object from the firm that sets the lower

tion earns no profits in equilibrium, whereas the firm with private information earns a positive profit in expectation. In our paper, we consider an asymmetric first-price private-value auction, which results in a quite different equilibrium structure.

[4]Studies dealing with strategic R&D or strategic entry is abundant. For example, see Spence (1984) [21], d'Aspremont and Jacquemin (1988) [3], Suzumura (1992) [23], Ishida et al. (2011) [8], Spence (1977) [20], and Dixit (1979) [4].

price.[5] Therefore, for all $i, j \in \{U, I\}$, $i \neq j$, if firm i that sets a price p_i lower than that of its rival's price p_j obtains a profit of $p_i - c_i$. Note that firm i obtains $(p_i - c_i)/2$ when $p_i = p_j$.

There is no pure strategy equilibrium in this game[6]. Hence, we discuss two classes of mixed strategy equilibria in this example.

Equilibrium 1 Let us consider a strategy profile in which firm U and firm I of type 1 randomize their prices on an interval $[(4 + c_U)/3, 2]$ and firm I of both type 2 and 3 set its price above 2. Specifically, firm U randomizes its price p_U according to the cumulative distribution function $P_U(p_U) = 1 - [(c_U + 1)/(3(p_U - 1))]$ on $[(4 + c_U)/3, 2)$ with a probability mass $(c_U + 1)/3$ at $p_U = 2$. Firm I of type 1 randomizes its price p_1 according to the cumulative distribution function $P_1(p_1) = 3 - 2(2 - c_U)/(p_1 - c_U)$ on $[(4 + c_U)/3, 2]$, and firm I of type $c_I \in \{2, 3\}$ randomizes their prices, p_2 and p_3, according to the uniform distribution function on $[2, 2 + \varepsilon]$.

Under the above strategy profile, firm U obtains a profit of $(4-2c_U)/3 > 0$ by setting its price in $[(4 + c_U)/3, 2]$, firm I of type 1 obtains a profit of $(1 + c_U)/3$ by setting its price in $[(4 + c_U)/3, 2)$, and firm I of type $c_I \in \{2, 3\}$ obtains zero profit. Obviously, firm I has no incentive to deviate.[7] Blume (2003)[1] shows that for small enough $\varepsilon > 0$, the firm U's expected profit is decreasing in $p_U \in [2, 2 + \varepsilon]$. Hence, the given strategy profile constitutes an equilibrium.

Equilibrium 2 We focus on the following strategy profile in which firm U and firm I of type $c_I \in \{1, 2\}$ randomize their prices on an interval $[1 + 2c_U/3, 3]$, and firm I of type 3 sets its price above 3. More specifically, firm U randomizes its price according to the cumulative distribution function

$$P_U(p_U) = \begin{cases} 1 - \frac{2c_U}{3(p_U-1)} & \text{for } p_U \in [\frac{3+2c_U}{3}, \frac{c_U+3}{2}], \\ 1 - \frac{2c_U(c_U-1)}{3(c_U+1)(p_U-2)} & \text{for } p_U \in [\frac{c_U+3}{2}, 3), \end{cases}$$

with a probability mass $2c_U(c_U - 1)/[3(c_U + 1)]$ at $p_U = 3$. Firm I of type 1 randomizes its price according to the cumulative distribution function $P_1(p_1) = 3[1 - (3 - c_U)/\{3(p_1 - c_U)\}]$ on $[1 + 2c_U/3, (c_U + 3)/2]$; firm I of type 2 randomizes its price according to the cumulative distribution function $P_2(p_2) = 3[1 - 1/3 - (3 - c_U)/\{3(p_2 - c_U)\}]$ on $[(c_U + 3)/2, 3]$; and firm I of type 3 randomizes its price according to the uniform distribution function on $[3, 3+\varepsilon]$. Similar to the case of Equilibrium 1, we can easily make sure that the given strategy profile constitutes an equilibrium. In this equilibrium, firm U obtains a profit of $(3 - c_U)/3$, firm I

[5]We can interpret this situation as a first-price sealed-bid procurement auction with one-sided cost uncertainty. Vickrey (1961)[24] investigates a similar situation under assumption of a continuous type space and uniform distribution.

[6]We provide a general proof of this result in our main theorem 4.1 where we outline that we can only obtain mixed strategy equilibria in this game.

[7]Setting $p_1 = 2$ is not optimal for firm I of type 1. However, the probability with which it sets $p_1 = 2$ is zero. Therefore, P_1 is still the best response to P_U.

of type 1 obtains a profit of $2c_U/3$, and firm I of type 2 obtains a profit of $2c_U(c_U - 1)/[3(c_U + 1)]$.

As will be appreciated from the foregoing, there are multiple equilibria in this model. An important point to note here is that in Equilibrium 1, firm I of type 3 sets a price lower than its cost (which is dominated by marginal cost pricing strategy by the same firm since such strategy can yield 'zero' profit instead of negative profit), whereas in Equilibrium 2, all firm prices are above their costs. Needless to say, if we focus on undominated equilibria such that no firm sets its price below its marginal cost, we obtain a unique equilibrium outcome (Theorem 4.1). In our example in this section 2.1, all undominated equilibria are outcome equivalent to Equilibrium 2.

2.2. A cost-reducing R&D by an entrant involving technological uncertainty

We present this example in the classical incumbent-entrant game with price being the strategic variable. We suppose that firm I is an entrant and firm U is the incumbent in a monopolistic market. Before entering the market, firm I decides on how much it invests in a cost-reducing R&D. Firm I's production cost can take two values, low-cost ($c_I = 1$) or high-cost ($c_I = 2$), and its realization depends on the amount of firm I's investment $C_I \in \{0, C_L, C_H\}$ where $0 < C_L < C_H$. If firm I chooses $C_I = 0$, its cost is 2, and if it chooses $C_I = C_H$, its cost is 1. Otherwise, c_I is drawn from $\{1, 2\}$ with equal probability. Firm U observes C_I but not the realized cost of c_I. Firm U's cost and the market demand function remain the same as in section 2.1. Now we examine each subgame of this game in order to determine the equilibira.

After firm I chooses $C_I = C_H$, firm U and firm I with cost $c_I = 1$ engage in Bertrand competition. In this subgame, there is a unique undominated equilibrium outcome where firm I serves the entire market demand at price $p_1 = c_U$. Firm I obtains a profit of $c_U - 1 - C_H$, whereas firm U earns zero profit. Similarly, if firm I chooses $C_I = 0$, then firm U serves the entire market demand at price $p_U = 2$, in which case firm I earns zero profit.

In the subgame in which firm I chooses $C_I = C_L$, we obtain a unique undominated equilibrium outcome. In this equilibrium, firm U sets its price according to the cumulative distribution function $P_U(p_U) = 1 - [c_U/2(p_U - 1)]$ on $[1 + c_U/2, 2)$ with a probability mass $c_U/2$ at $p_U = 2$. Firm I of type 1 randomizes its price according to the cumulative distribution function $P_1(p_1) = 2 - (2 - c_U)/(p_1 - c_U)$ on $[1 + c_U/2, 2]$, and firm I of type 2 randomizes its price p_2 according to the uniform distribution function on $[2, 2 + \varepsilon]$.[8] Therefore, firm I's *ex ante* expected profit after choosing $C_I = C_L$ is given by $c_U/4$.

[8]In other equilibria, firm I of type 2 sets a price lower than its marginal cost. For details, see Theorem 4.1.

Focusing on the equilibrium such that no firm sets a price below its marginal cost both on and off the equilibrium path, we obtain a unique prediction: if $c_U - 1 - C_H > \max\{c_U/4 - C_L, 0\}$, then firm I chooses C_H; if $c_U/4 - C_L > \max\{c_U - 1 - C_H, 0\}$, then firm I chooses C_L; otherwise, firm I enters the market without making any R&D investment. In this case, the condition under which firm I would prefer setting investment level $C_I = C_H$ to $C_I = C_L$ could be simplified into

$$\frac{3}{4}c_U - 1 > C_H - C_L.$$

Interestingly, if $c_U < 4/3$, firm I would never choose the most effective R&D investment even when C_H is almost[9] equal to zero. Choosing $C_I = C_H$ (≈ 0) is the most efficient investment for firm I in terms of cost reduction. However, such an investment will induce a harsh price competition leading to loss of profit for firm I. Therefore, in order to avoid such prospective loss of profit, firm I would not want invest at C_H level.

Note that we obtain a similar result when we assume $c_U \leq c_1$. In this case, firm I would never choose $C_I = C_H$ since it could not earn a positive profit if firm U could correctly estimate the exact value of c_I. However, if $c_U/4 > C_L$, firm I would choose $C_I = C_L$ and obtain a positive expected profit. This implies that firm I will be able to recover from its technological disadvantage by building an informational advantage with an inefficient level of investment. In section 5, we provide further discussion related to the results in this example.

3. The model

Two firms, indexed by $i, j \in N = \{U, I\}$, are engaged in a Bertrand price competition with homogeneous goods and equal rationing rule. Firms that set the lowest price serve the whole market demand. In case of a tie, firms share the demand equally. Hence, firm i faces a market demand of

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j, \\ D(p_i)/2 & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j. \end{cases}$$

The demand function $D : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies the following properties. First, there exists a "choke-off price" $p^{\max} \in \mathbb{R}_{++}$ such that $D(p) > 0$ for all $p \in (0, p^{\max})$, and $D(p) = 0$ for all $p \in (p^{\max}, +\infty)$. Second, D is continuously differentiable on $(0, p^{\max})$ with the property that $D'(p) \leq 0$ and left-continuous at $p^{\max}$.

[9] We use the word 'almost' here to emphasize that in our game the incumbent makes its pricing decisions after observing the entrant's investment level. Since a 'zero' investment decision may likely eliminate such a conditioning by the incumbent, any positive investment close to 'zero' would suffice to retain this stage which plays an important role in our equilibrium characterization. In section 5.2, we provide further discussion on the observability of the entrant's investment level.

Firm U's marginal unit cost is publicly known to be c_U. Firm I, however, privately observes its own marginal unit cost c_I which can take K-values $\{c_1, \ldots, c_K\}$ with probability $\eta_k \in (0, 1)$ for $k \in \{1, \ldots, K\}$, respectively. We assume that $0 < c_1 < \cdots < c_K < p^{\max}$, and firm I is of type k if it's marginal cost $c_I = c_k$. If $c_K < c_U$, there are mixed strategy equilibria that are essentially equivalent to equilibria characterized in Blume (2003)[1]. Hence, we assume that $c_U < c_K$.[10]

The profit function of firm i is given by $\pi_i(p_i, p_j) = (p_i - c_i)D_i(p_i, p_j)$ for each $(p_i, p_j) \in \mathbb{R}^2_+$. We assume that $(p_i - c_i)D(p_i)$ is single peaked in price p_i on $(0, p^{\max}]$. Let p^m_l denote the monopoly price for each cost realization: $p^m_l = \operatorname{argmax}_{p \in \mathbb{R}_+}(p_l - c_l)D(p_l)$ for each $l \in \{U, 1, \ldots, K\}$. To ensure that no firm can enjoy the monopoly profit, we assume that $\min\{p^m_1, p^m_U\} > c_K$.

A (mixed) strategy for firm U is a probability distribution P_U on $\mathbb{R}_+$. A (behavior) strategy for firm I is denoted by P_I which specifies a probability distribution on $\mathbb{R}_+$ depending on its type k. Thus, we denote by P_k the probability distribution with which firm I of type k sets its price.

3.1. The undominated equilibria

The Equilibrium Concept we use in this analysis is the *Bayesian Nash equilibrium* (BNE or equlibrium). Hereafter, (P^*_U, P^*_I) denotes an equilibrium strategy profile where $P^*_I \equiv (P^*_1, \ldots, P^*_K)$. We denote the expected equilibrium profit of firm U and firm I of type k by Π^*_U and Π^*_k, respectively. Let $\underline{p}_l$ and $\overline{p}_l$ be the lowest and highest element in $\operatorname{supp}(P^*_l)$ for $l \in \{U, 1, \ldots, K\}$, where $\operatorname{supp}(P)$ denotes the support of the probability distribution P.[11]

Clearly, it is a weakly dominated strategy for each firm to set its price strictly lower than its marginal cost. Therefore, we characterize equilibria with the following refinement.

Refinement 1 (weak dominance). $\underline{p}_l \geq c_l$ *for all* $l \in \{U, 1, \ldots, K\}$.

If firm I follows a strategy according to Refinement 1, firm U can be sure to get as much as $\overline{\Pi}_U(k) \equiv (1 - \sum_{\ell<k} \eta_\ell)(c_k - c_U)D(c_k)$ by setting its price slightly lower than c_k. Note that $\overline{\Pi}_U(1)$ is defined as $(c_1 - c_U)D(c_1)$. Therefore, in any equilibrium with Refinement 1, firm U's equilibrium payoff Π^*_U is no less than the maximum of $\overline{\Pi}_U(k)$. Since $c_U < c_K$, the fact that $\overline{\Pi}_U(k) \leq (c_1 - c_U)D(c_1)$ for all k implies that $c_U < c_1$. Furthermore, if this inequality, $\overline{\Pi}_U(k) \leq (c_1 - c_U)D(c_1)$, holds for all k, the undominated equilibrium induces the conventional outcome

[10] Note that, if $c_K < c_U$, there exists an equilibrium such that firm I serves the entire market at a price equal to c_U. If $c_U = c_K$, there exists no equilibrium in this model. This result can be proved in the same manner as in Example 2 of Maskin and Riley (2000) [13]. The above facts and Theorem 4.1 in Section 4 implies that there always exists at least one equilibrium if $c_U \neq c_K$.

[11] Precisely, $\operatorname{supp}(P)$ is defined as the set of all points p in $\mathbb{R}_+$ for which every open neighbourhood of p has positive measure on the probability space $(\mathbb{R}_+, \mathbb{B}(\mathbb{R}_+), P)$, where $\mathbb{B}(\mathbb{R}_+)$ is the Borel algebra on a set $\mathbb{R}_+$. Hence, under the given P^*_l, for all $\epsilon > 0$, $\Pr(p_l \in [\underline{p}_l, \underline{p}_l + \epsilon)\}) > 0$ and $\Pr(p_l \in (\underline{p}_l - \epsilon, \underline{p}_l)) = 0$, and similarly, $\Pr(p_l \in (\overline{p}_l - \epsilon, \overline{p}_l]) > 0$ and $\Pr(p_l \in (\overline{p}_l, \overline{p}_l + \epsilon)) = 0$.

of firm U serving the entire market at a price c_1.[12] Therefore, we assume that $\overline{\Pi}_U(k) > (c_1 - c_U)D(c_1)$ for some $k > 1$.

Assumption 1. $\overline{\Pi}_U(k) \equiv (1 - \sum_{\ell<k} \eta_\ell)(c_k - c_U)D(c_k) > (c_1 - c_U)D(c_1)$ *for some* $k > 1$.

Note that if $c_U \geq c_1$, this assumption immediately holds.

Hereafter, $\bar{k}$ denotes the minimum element in firm I's types that maximize $\overline{\Pi}_U(k)$.

Definition 3.1. $\bar{k} \equiv \min\{\arg\max_k \overline{\Pi}_U(k)\}$.

In this study, we obtain the virtually unique undominated equilibrium with intuitive properties such that $\Pi^*_U = \overline{\Pi}_U(\bar{k})$, $\Pi^*_k > 0$ for $k < \bar{k}$, and $\Pi^*_k = 0$ for $k \geq \bar{k}$.

4. Main results

To start with, we first provide a preliminary result regarding the equilibrium profit of firm I. Fix an equilibrium (P^*_U, P^*_I) and suppose that firm I of type k' obtains a positive equilibrium profit, which implies that $\bar{p}_U > c_{k'}$ under the given P^*_U. Then, firm I of type $k < k'$ can enjoy a positive profit by setting $p_k = c_{k'}$. Therefore, if there exists an equilibrium such that $\Pi^*_{k'} > 0$, it must satisfy $\Pi^*_k > 0$ for all $k < k'$.

The above discussion is summarized in Remark 4.1.

Remark 4.1. *In any equilibrium, if* $\Pi^*_{k'} > 0$ *for* $k' > 1$, *then* $\Pi^*_k > 0$ *for all* $k \leq k'$.

Let k^*_+ denote the maximum type of firm I that earns a positive expected profit, and let k^*_0 denote the minimum type of firm firm I that earns zero profit in (P^*_U, P^*_I). Remark 4.1 shows that $k^*_+ + 1 = k^*_0$.

The following theorem provides us with a necessary condition for a given strategy profile to constitute an undominated equilibrium. It also shows that the equilibrium outcome is unique.[13]

Theorem 4.1. *In any equilibrium that satisfies Refinement 1, the following properties hold:*

1. $k^*_+ + 1 = \bar{k}$,
2. *the lower and upper bounds of the support of the equilibrium strategies satisfy that for* $k \in \{1, \ldots, k^*_+\}$

$$\underline{p}_U = \underline{p}_1,\ \bar{p}_U = \bar{p}_{k^*_+} = c_{k^*_+ + 1},\ \textit{and}\ \bar{p}_k = \underline{p}_{k+1}; \tag{1}$$

$$\overline{\Pi}_U(k^*_+ + 1) = \left\{1 - \sum_{\ell=1}^{k} \eta_\ell\right\} (\bar{p}_k - c_U)D(\bar{p}_k) = (\underline{p}_U - c_U)D(\underline{p}_U); \tag{2}$$

[12] See Remark 6.1 in Appendix A.

[13] Technically, there is a continuum of equilibria all of which are outcome equivalent to the equilibrium described in Theorem 4.1.

3. *the equilibrium strategies satisfy the followings:*

   - *firm U randomizes its price according to the cumulative distribution function*

$$P_U^*(p_U) = 1 - \frac{\Pi_{\ell=1}^{k}(\underline{p}_\ell - c_\ell)}{\Pi_{\ell=2}^{k}(\underline{p}_\ell - c_{\ell-1})} \times \frac{D(\underline{p}_U)}{(p_U - c_k)D(p_U)}, \tag{3}$$

   for $p_U \in [\underline{p}_k, \overline{p}_k)$ *and* $k \in \{1, \ldots, k_+^*\}$,[14] *and* P_U^* *has a probability mass* $\Pi_{\ell=1}^{k_+^*}(\underline{p}_\ell - c_\ell)D(\underline{p}_U)/\Pi_{\ell=2}^{k_+^*}(\underline{p}_\ell - c_{\ell-1})(c_{k_+^*+1} - c_{k_+^*})D(c_{k_+^*+1})$ *at* $p_U = c_{k_+^*+1}$,

   - *firm I of type* $k \in \{1, \ldots, k_+^*\}$ *randomizes its price according to the cumulative distribution function*

$$P_k^*(p_k) = \frac{1}{\eta_k}\left[1 - \sum_{\ell=1}^{k-1}\eta_\ell - \frac{(1 - \sum_{\ell=1}^{k_+^*}\eta_\ell)(c_{k_+^*+1} - c_U)D(c_{k_+^*+1})}{(p_k - c_U)D(p_k)}\right], \tag{4}$$

   on $[\underline{p}_k, \overline{p}_k]$.

Proof. See Appendix A. □

Theorem 4.1 shows that in the undominated equilibrium, firm U's expected profit is given by the supremum value that firm U can be sure to get when firm I follows marginal cost pricing; that is, $\Pi_U^* = \overline{\Pi}_U(\overline{k}) = \max_k \overline{\Pi}_U(k)$. The maximum price firm U sets with a positive probability is $\overline{p}_U = c_{\overline{k}}$. Hence, firm I of type $k < \overline{k}$ earns a positive expected profit even when $c_k > c_U$. This result implies that firm I can obtain an information rent owing to its information advantage.

We now briefly explain properties of the equilibrium strategies characterized in the theorem 4.1. In this equilibrium, firm U and firm I of type $k \leq k_+^*$ obtain positive profits. This implies that the lowest prices that these firms will set must be strictly higher than their respective marginal costs: $\underline{p}_l > c_l$ for $l \in \{U, 1, \ldots, k_+^*\}$. The conditions in the first and second statements of the theorem 4.1 ensure that this is true. Moreover, since firm I of type $k \geq k_0^*$ will not be able to obtain a positive profit, the probability with which firm U sets its price higher than the price set by firm I of type $k \geq k_0^*$ is zero, i.e., $\overline{p}_U \leq \underline{p}_k$ for $k \geq k_0^*$.

In this equilibrium, firm U cannot make different types of firm I to be indifferent among the same range of prices. Given firm U's pricing strategy, firms I's (expected) profit function has a single crossing property; that is, the cross partial derivative of firm I's profit function with respect to its price and marginal cost is positive. Therefore, we must have the property that $\overline{p}_k \leq \underline{p}_{k'}$ for $k < k'$.[15] Firm U randomizes its price on $[\underline{p}_U, \overline{p}_U] = [\underline{p}_1, \overline{p}_1] \cup \cdots \cup [\underline{p}_{k_+^*}, \overline{p}_{k_+^*}]$ to

[14]Note that $\Pi_{\ell=2}^{k}(\underline{p}_\ell - c_{\ell-1}) \equiv 1$ for $k = 1$. Since $\overline{p}_k = \underline{p}_{k+1}$ for $k \leq k_+^* - 1$, the c.d.f. P_U^* is continuous on $[\underline{p}_U, \overline{p}_U)$.

[15]For more details, see the proof of Lemma 6.4 in Appendix A.

make firm I of type $k \le k_+^*$ to be indifferent among all prices in the set $[\underline{p}_k, \overline{p}_k]$. The second statement of the theorem are induced from conditions for firm U to be indifferent at boundaries of the partition: $\{\underline{p}_U, \underline{p}_2, \underline{p}_3, \dots, \overline{p}_U\}$.

To ensure that firm I of type $k \le k_+^*$ obtains a positive profit, firm U must choose a price to lose against firm I of type $k \le k_+^*$ with a positive probability; that is, firm U sets $\overline{p}_U$ with a positive probability. Firm I of type $k \le k_+^*$ randomizes its price on $[\underline{p}_k, \overline{p}_k]$ in such a manner that the marginal distribution of their pricing strategies ensures firm U to be indifferent among its prices in the range $[\underline{p}_U, \overline{p}_U]$.

As noted earlier, $\overline{p}_U$ is a losing price against firm I of type $k \le k_+^*$. If $\underline{p}_k > \overline{p}_U$ for any $k \ge k_0^*$, firm U has incentive to increase $\overline{p}_U$. Therefore, in any equilibrium we must be satisfy $\overline{p}_U = \underline{p}_{k'}$ for some $k' \ge k_0^*$.[16] Recall that firm I of type $k \ge k_0^*$ earns zero profit. Since our focus is on the equilibrium with Refinement 1, we obtain $\overline{p}_U = c_{k_+^*+1} \equiv c_{k_0^*} = \underline{p}_{k_0^*}$.

Existence of the undominated equilibrium. In what follows, we ensure that at least one equilibrium which meets the conditions in Theorem 4.1 exists. Consider a strategy profile (P_U^*, P_I^*) that satisfies all conditions in the theorem. In addition to this, let us suppose that firm I of type $\overline{k} = k_+^* + 1$ randomizes its price according to the uniform distribution on $[c_{k_+^*+1}, c_{k_+^*+1} + \varepsilon]$ and that firm I of type $k > \overline{k} = k_+^* + 1$ sets $p_k = c_k$ with probability one.

First we ensure that $\underline{p}_U, \underline{p}_k$, and $\overline{p}_k$ are well defined. Recall that $c_{k_+^*+1} \le c_K < p_U^m$. Therefore, $(p_U - c_U)D(p_U)$ is increasing in p_U on $(c_U, c_{k_+^*+1}]$. Since, moreover, $(1 - \sum_{\ell=1}^{k_+^*} \eta_\ell) \in (0, 1)$, the lower bound $\underline{p}_U$ that satisfies $(\underline{p}_U - c_U)D(\underline{p}_U) = (1 - \sum_{\ell=1}^{k_+^*} \eta_\ell)(c_{k_+^*+1} - c_U)D(c_{k_+^*+1})$ is uniquely derived in $(c_U, c_{k_+^*+1})$. Furthermore, since we suppose that

$$
\begin{aligned}
\overline{\Pi}_U(\overline{k}) &= \left\{1 - \sum_{\ell=1}^{k_+^*} \eta_\ell\right\} (c_{k_+^*+1} - c_U)D(c_{k_+^*+1}) \\
&> \max_{k<k_+^*} \left\{1 - \sum_{\ell=1}^{k} \eta_\ell\right\} (c_{k+1} - c_U)D(c_{k+1}) = \max_{k<k_+^*} \overline{\Pi}_U(k),
\end{aligned}
$$

we obtain $c_U < \underline{p}_1 < \overline{p}_1 < \overline{p}_2 < \cdots < \overline{p}_{k_+^*-1} < c_{k_+^*+1}$ and $c_{k+1} < \overline{p}_k = \underline{p}_{k+1}$ for all $k < k_+^*$. Note that for any $l \in \{U, 1, \dots, k_+^*\}$, the function P_l^* is a well-defined cumulative distribution function: P_l^* is non-decreasing, $P_l^*(\underline{p}_l) = 0$, and $P_l^*(\overline{p}_l) = 1$.

If firm U follows the prescribed strategy, firm I of type $k \in \{1, \dots, k_+^*\}$ obtains $\Pi_{\ell=1}^k(\underline{p}_\ell - c_\ell)D(\underline{p}_U)/\Pi_{\ell=2}^k(\underline{p}_\ell - c_{\ell-1})$ by setting $p_k \in [\underline{p}_k, \overline{p}_k]$. Hence, it is indifferent among all $p_k \in [\underline{p}_k, \overline{p}_k]$. Moreover, if $(p_k - c_k)D(p_k)(1 - P_U^*(p_k))$ stays constant on $p_k \in [\underline{p}_k, \overline{p}_k]$, then $(p_{\hat{k}} - c_{\hat{k}})D(p_{\hat{k}})(1 - P_U^*(p_{\hat{k}}))$ is increasing (decreasing) in $p_{\hat{k}} \in [\underline{p}_k, \overline{p}_k]$

[16]For the discontinuity of the firm U's expected profit function at $\overline{p}_U$ to be ruled out, $P_{k'}^*$ must not have a probability mass at $\overline{p}_U$.

for $\hat{k} > k$ ($\hat{k} < k$). Therefore, setting $p_k \in [\underline{p}_k, \overline{p}_k]$ is optimal for firm I of type $k \in \{1, \ldots, k_+^*\}$.[17]

Since $\overline{p}_U = c_{k_+^*+1}$, it is obvious that firm I of type $k > k_+^*$ has no incentive to deviate given the strategy profile. Therefore, we only need to make sure that P_U^* is the best response to the given pricing strategy of firm I. Since firm U randomizes its price on $[\underline{p}_U, \overline{p}_U]$, it must be indifferent among all $p_U \in [\underline{p}_U, \overline{p}_U]$. By setting $p_U \in [\underline{p}_k, \overline{p}_k]$, firm U obtains

$$\left[1 - \left\{\sum_{\ell=1}^{k-1} \eta_\ell + \eta_k P_k^*(p_U)\right\}\right](p_U - c_U)D(p_U) = \left[1 - \sum_{\ell=1}^{k_+^*} \eta_\ell\right](c_{k_+^*+1} - c_U)D(c_{k_+^*+1})$$
$$= \overline{\Pi}_U(\overline{k}).$$

Recall that firm I of type $k > \overline{k} \equiv k_+^* + 1$ sets $p_k = c_k$ with probability one. Given $\overline{k}$, we have $\Pi_U^* = \overline{\Pi}_U(\overline{k}) \geq \overline{\Pi}_U(k)$ for $k > \overline{k}$. Therefore, clearly, firm U has no incentive to set its price above $\overline{p}_U + \varepsilon$. Applying Blume (2003), the expected payoff of firm U is non-increasing in p_U on $[\overline{p}_U, \overline{p}_U + \varepsilon]$. Furthermore, firm U has no incentive to set its price less than $\underline{p}_U$. This proves that the given strategy profile (P_U^*, P_I^*) constitutes an undominated equilibrium.

Remark 4.2. *Under the given P_U^* characterized in Theorem 4.1, setting $\overline{p}_{k_+^*} = c_{k_+^*+1}$ is not optimal for firm I of type k_+^* because firm U sets $\overline{p}_U = c_{k_+^*+1}$ with a positive probability. However, the set $\{p_{k_+^*} = c_{k_+^*+1}\}$ has measure zero on a given probability space endowed with $P_{k_+^*}^*$. Hence, $P_{k_+^*}^*$ is the best response to P_U^* even though it includes $\overline{p}_{k_+^*} = \overline{p}_U = c_{k_+^*+1}$ in its support.*

Given $\overline{p}_U = c_{k_+^*+1}$, the equilibrium strategies of firm U and firm I of type $k \leq k_+^*$ are uniquely derived from conditions (1)–(4). Therefore, Theorem 4.1 shows the uniqueness of the undominated equilibrium outcome; that is, the probability distribution of market price and the firms' expected equilibrium profits are uniquely determined.

4.1. Discussion: Almost complete information

Here, we consider the situation in which cost uncertainty almost disappears. The following Proposition 4.1 shows that the outcome of the undominated equilibrium converges to the conventional outcome, where the low-cost firm serves the entire market at a price equal to the high marginal cost as cost uncertainty is reduced.

Proposition 4.1.
- *In the undominated equilibrium, as $\eta_{\tilde{k}}$ converges to one for some $\tilde{k}$ such that $c_{\tilde{k}} > c_U$, firm U almost surely serves the whole market demand at a price approximately equal to $c_{\tilde{k}}$.*

[17]In particular, $\overline{p}_{k_+^*}$ is not the optimal price for firm I of type k_+^*. However, since the event $\{p_{k_+^*} = \overline{p}_{k_+^*}\}$ has measure zero under $P_{k_+^*}^*$, this fact does not induce any problem. See Remark 4.2.

- *Suppose that $c_1 < c_U$. In this case, as $\eta_{\tilde{k}}$ converges to one for some $\tilde{k}$ such that $c_{\tilde{k}} \leq c_U$, firm I almost surely obtains an expected profit of approximately equal to $c_U - c_{\tilde{k}}$.*

Proof. Let us suppose that $\eta_{\tilde{k}}$ is almost equal to 1 for some $\tilde{k}$ such that $c_{\tilde{k}} > c_U$. Then, Theorem 4.1 shows that $k^*_+ = \tilde{k} - 1$ and $\overline{p}_U = c_{\tilde{k}}$. Since $P_{\tilde{k}}(c_{\tilde{k}}) = 0$, the marginal probability distribution of firm I's strategy sets p_I above $c_{\tilde{k}}$ almost surely. Moreover, by Equation (2) in Theorem 4.1, we obtain $(1-\epsilon)(c_{\tilde{k}}-c_U)D(c_{\tilde{k}}) = (\underline{p}_U - c_U)D(\underline{p}_U)$; that is, $\underline{p}_U \approx c_{\tilde{k}}$. Therefore, as $\eta_{\tilde{k}}$ converges to one for $\tilde{k}$ such that $c_{\tilde{k}} > c_U$, in the undominated equilibrium, firm U serves the whole market at price $p_U \approx c_{\tilde{k}}$ almost surely.

Next, suppose that $\eta_{\tilde{k}}$ is almost equal to 1 for some $\tilde{k}$ such that $c_{\tilde{k}} \leq c_U$. Then, we obtain k^*_+ as the maximum k such that $c_k \leq c_U$. Theorem4.1 shows that $\overline{p}_U = \overline{p}_{k^*_+} = c_{k^*_+ +1}$. Equation (2) in Theorem 4.1 shows that $\underline{p}_U$ and $\underline{p}_{\tilde{k}}$ are almost equal to c_U. Moreover, $P^*_U(\underline{p}_{\tilde{k}})$ is almost equal to 0. Then, firm I of type $\tilde{k}$ obtains an expected profit of approximately equal to $c_U - c_{\tilde{k}}$ in the undominated equilibrium. These results imply that as $\eta_{\tilde{k}}$ converges to one for some $\tilde{k}$ such that $c_{\tilde{k}} \leq c_U$, firm I is type $\tilde{k}$ with almost probability one and obtains an expected profit of approximately equal to $c_U - c_{\tilde{k}}$. □

5. Application: Cost-reducing R&D investment by an entrant

In this section, we apply our results obtained in section 4 to analyze the investment strategies discussed in section 2.2. Suppose that firm I is an entrant with the initial unit cost c_2. Before entering the market, firm I makes an investment, which affects the success rate of a cost-reducing R&D undertaken by firm I. If firm I succeeds in the R&D, its marginal cost will be reduced to c_1, where $c_1 < c_2$. More specifically, if firm I invests $\gamma C(\eta)$, its marginal cost is drawn from $\{c_1, c_2\}$ with probabilities $(\eta_1, \eta_2) \equiv (\eta, 1-\eta)$ where $\gamma > 0$ is a constant parameter. The investment function of R&D, $C(\cdot)$, satisfies $C'(\eta) > 0$ and $C''(\eta) \geq 0$ for all $\eta > 0$. For simplification of analysis, we assume that $C(0) = 0$, $\lim_{\eta \to 0} C'(\eta) = 0$, and $\lim_{\eta \to 1} C'(\eta) = +\infty$.[18]

We assume that Firm U (the incumbent monopolist)'s marginal cost c_U is in $[c_1, c_2)$ and that firm U observes firm I's choice of $\gamma C(\eta)$, but not the realized value of c_I. After firm I has made its investment and has realized the resulting cost, both firms engage in a Bertrand competition.

In this section, we analyze the perfect Bayesian equilibria (PBEs) of the following game:

1. Firm I invests $\gamma C(\eta)$, and firm U observes it. Only firm I observes the realized value of its marginal cost.

[18]We use these assumptions for eliminating the possibility of a corner solution of the firm I's optimization problem; however, they are not essential for our results to hold.

2. Firms engage in a Bertrand competition.

We focus on PBEs in which firms never set their price below their marginal costs both on and off the equilibrium path. Hereafter, we call such equilibria *undominated equilibria*.

5.1. The optimal cost-reducing R&D

Once firm I invests $\gamma C(\eta) > 0$ for $\eta \in (0,1)$, by Theorem 4.1, we obtain that $k_+^* = 1$ and $\overline{p}_U = c_2$. We also find the lower bound $\underline{p}_U(\eta)$ by the solving Equation $(1-\eta)(c_2-c_U)D(c_2) = (\underline{p}_U - c_U)D(\underline{p}_U)$. Note that $\underline{p}_U(\eta)$ is decreasing in η, $\underline{p}_U(0) = c_2$, and $\underline{p}_U(1) = c_U$.

Firm U's pricing strategies under an undominated equilibrium is given by

$$P_U(p_U) = 1 - \frac{(\underline{p}_U(\eta) - c_1)D(\underline{p}_U(\eta))}{(p_U - c_1)D(p_U)},$$

on $[\underline{p}_U(\eta), c_2)$ with a probability mass $(\underline{p}_U(\eta) - c_1)D(\underline{p}_U(\eta))/(c_2 - c_1)D(c_2)$ at c_2.

Firm I's *ex ante* expected profit from choosing η is given by

$$\Pi_I(\eta|\gamma) = \eta(\underline{p}_U(\eta) - c_1)D(\underline{p}_U(\eta)) - \gamma C(\eta).$$

The first-order condition is

$$(\underline{p}_U(\eta) - c_1)D(\underline{p}_U(\eta)) + \eta \underline{p}_U'(\eta)\left[D(\underline{p}_U(\eta)) + D'(\underline{p}_U(\eta))(\underline{p}_U(\eta) - c_1)\right] - \gamma C'(\eta) = 0, \tag{5}$$

where $\underline{p}_U'(\eta) = -(c_2 - c_U)D(c_2)/[D(\underline{p}_U(\eta)) + D'(\underline{p}_U(\eta))(\underline{p}_U(\eta) - c_U)]$.[19] We assume that $D''(p)D(p) - 2\{D'(p)\}^2 \leq 0$ for $p \in [c_U, c_2]$, which is sufficient for the second order condition to hold.[20]

Because of the concavity of $\Pi_I(\cdot)$, the fact that $\Pi_I'(0|\gamma) = (c_2 - c_1)D(c_2) > 0$ and $\lim_{\eta\to 1} C'(\eta) = +\infty$, the optimal $\eta^*(\gamma)$ is uniquely determined in $(0,1)$, and it is continuous and strictly decreasing in $\gamma > 0$.[21]

Proposition 5.1. *If* $\Pi_I'(\eta = 1|\gamma = 0) < 0$, *then* $\lim_{\gamma\to 0} \eta^*(\gamma) = \eta^*(0) < 1$; *otherwise,* $\lim_{\gamma\to 0} \eta^*(\gamma) = 1$.

Note that $\eta^*(0)$ is the solution to the equation $\Pi_I'(\eta|\gamma = 0) = 0$ and that

$$\Pi_I'(\eta = 1|\gamma = 0) = (c_U - c_1)D(c_U) - (c_2 - c_U)D(c_2)\frac{D(c_U) + D'(c_U)(c_U - c_1)}{D(c_U)}.$$

[19]This equation is induced by the implicit function theorem.

[20]See Appendix B.

[21]By the implicit function theorem, we obtain

$$\frac{d\eta^*}{d\gamma} = \frac{C'(\eta^*)}{\Pi_I''(\eta^*)} < 0.$$

Proof. Firm I's maximizing problem always has an interior solution $\eta^*(\gamma) \in (0,1)$ for $\gamma > 0$, and $\eta^*(\gamma)$ is continuously decreasing in $\gamma > 0$. Moreover, for any $\eta < 1$, we can obtain $\gamma C'(\eta)$ arbitrarily close to 0 by choosing a small enough γ. Therefore, $\eta^*(\gamma)$ converges to $argmax_{\eta\in[0,1]}\Pi_I(\eta|\gamma = 0)$ as γ approaches 0. Hence, if $\Pi_I'(\eta = 1|\gamma = 0) < 0$, $\lim_{\gamma\to 0}\eta^*(\gamma) = \eta^*(0) < 1$. Otherwise, $\lim_{\gamma\to 0}\eta^*(\gamma) = 1$. □

Proposition 5.1 shows that if c_U is close to c_1, $\Pi_I'(\eta = 1|\gamma = 0) < 0$ and $\lim_{\gamma\to 0}\eta^*(\gamma) = \eta^*(0) < 1$. If c_U is close to c_2, then $\Pi_I'(\eta = 1|\gamma = 0) > 0$ and $\lim_{\gamma\to 0}\eta^*(\gamma) = \eta^*(0) = 1$. This result upholds our intuition. Recall that if firm I invests $\gamma C(1)$, then its marginal cost is c_1 and it serves the whole market at a price c_U. Therefore, if c_U is close to c_1, the productive R&D investment induces harsh competition, which results in almost zero profit for firm I. Hence, to avoid such intense competition, firm I intentionally chooses inefficient investment even though it can conduct the most productive R&D with less investment.

5.2. On the observability of the cost of R&D firm I made

In this subsection, we assume that $\gamma C(\eta)$ is unobservable for firm U. In this case, firm I's deviation with respect to $\gamma C(\eta)$ does not affect firm U's pricing strategy. That is, if firm U believes that firm I invests $\gamma C(\eta^*)$, firm U randomizes its price according to

$$P_U(p_U) = 1 - \frac{(\underline{p}_U^* - c_1)D(\underline{p}_U^*)}{(p_U - c_1)D(p_U)},$$

on $[\underline{p}_U^*, c_2)$ with a probability mass $(\underline{p}_U^* - c_1)D(\underline{p}_U^*)/(c_2 - c_1)D(c_2)$ at c_2. Note that $\underline{p}_U^*$ is the solution to the equation $(1-\eta^*)(c_2 - c_U)D(c_2) = (\underline{p}_U^* - c_U)D(\underline{p}_U^*)$. Firm I's expected profit is given by $\eta(\underline{p}_U(\eta^*) - c_1)D(\underline{p}_U(\eta^*)) - \gamma C(\eta)$. Hence, firm I's optimal η^* is characterized by the following simultaneous equations

$$(\underline{p}_U^* - c_1)D(\underline{p}_U^*) - \gamma C'(\eta^*) = 0 \tag{6}$$

$$(1-\eta^*)(c_2 - c_U)D(c_2) = (\underline{p}_U^* - c_U)D(\underline{p}_U^*). \tag{7}$$

Let $\underline{p}_{un}^*$ and η_{un}^* denote the solution to (6)–(7); similarly, let $\underline{p}_o^*$ and η_o^* denote the solution to firm I's optimization problem in the previous subsection.

Proposition 5.2. *$\eta_o^* < \eta_{un}^*$ and $\underline{p}_{un}^* < \underline{p}_o^*$.*

Proof. Substituting $\underline{p}_{un}^*$ and η_{un}^* into (5), we obtain

$$\eta_{un}^*\underline{p}_U'(\eta_{un}^*)\left[D(\underline{p}_{un}^*) + D'(\underline{p}_{un}^*)(\underline{p}_{un}^* - c_1)\right] < 0, \tag{8}$$

The negative sign follows from the facts that the lower bound of firm U's pricing strategy is decreasing in η and $D(p)(p - c_1)$ is increasing in $p \in [c_1, c_2]$. Hence, we obtain $\eta_o^* < \eta_{un}^*$ and $\underline{p}_{un}^* < \underline{p}_o^*$. □

The inequality (8) shows that if firm U observes $\gamma C(\eta)$, firm I can increase its profit by choosing less productive investment compared to the case in which firm U does not observe $\gamma C(\eta)$. Therefore, firm I has an incentive to disclose its level of investment on the cost-reducing R&D to firm U.

5.3. An incentive to enter the market when the incumbent possesses technical advantages

In this subsection, we consider the case in which $c_U < c_1 < c_2$. This assumption captures the situation where the incumbent possesses technical advantages which comes from proprietary technology and accumulated knowledge. If firm U does not face uncertainty about firm I's cost, firm I cannot earn a positive profit, and hence, it has no incentive to operate in the market. The following discussion shows that inducing a cost uncertainty appropriately, firm I can always enjoy a positive expected profit even when it cannot achieve the marginal cost lower than that of firm U.

Under the assumption of $c_1 > c_U$, if firm I chooses η such that $(c_1 - c_U)D(c_1) \geq (1-\eta)(c_2-c_U)D(c_2)$, the undominated equilibrium induces the outcome of firm U serving the entire demand at price c_1. This inequality can be simplified into $\eta \geq 1 - (c_1 - c_U)D(c_1)/(c_2 - c_U)D(c_2) \equiv \tilde{\eta}$. If firm I chooses $\eta \in (0, \tilde{\eta})$, it obtains an expected profit of $\Pi_I(\eta) = \eta(\underline{p}_U(\eta) - c_1)D(\underline{p}_U(\eta)) - \gamma C(\eta)$. Note that $\underline{p}_U(0) = c_2$ and $\underline{p}_U(\tilde{\eta}) = c_1$. Since $\Pi_I'(0) > 0$ and $\Pi_I(\tilde{\eta}) < 0$, we always have an optimal $\eta^* \in (0, \tilde{\eta})$.[22] That is, entering the market after conducting R&D with η^* allows firm I to earn a positive expected profit even when it faces a second-mover disadvantage.

Remark 5.1. *In this section, the initial cost of firm I is assumed to be c_2. Hence, its R&D results in cost reduction in terms of expected value. A point to note here is that an R&D inducing a deterioration of technology can be better than a situation with no investment. To see this, assume that the initial cost of firm I is $c_1 > c_U$. Let us suppose that if firm I spends on $\gamma C(\eta_2)$, firm I's marginal cost increase to c_2 with a probability η_2. In the same manner as the above, we can make sure the fact that by choosing an appropriate $\eta_2^* \in (0, 1)$, firm I can secure a positive expected profit in the price competition stage. Interestingly, although its investment induces a deterioration of technology in the sense that investing increases the probability with which $c_I = c_2 > c_1$, if γ is small enough, firm I can obtain a positive expected profit.*

6. Concluding remarks

The main finding of this study is that in a Bertrand duopoly with one-sided cost uncertainty, all undominated equilibria induce a unique equilibrium outcome. In the undominated equilibria, firm U's expected profit is determined by the

[22] For the second-order condition to be satisfied, we assume that $D''(p)D(p) - 2\{D'(p)\}^2 \leq 0$ for $p \in [c_U, c_2]$.

highest profit that firm U is guaranteed to get when firm I follows marginal cost pricing. Exploiting the uniqueness of firm U's equilibrium profit, we obtain a virtually unique undominated equilibrium.

The uniqueness of the undominated equilibria in our model allows us to apply our model to investigate various economic situations. In this paper, we apply our results to examine the effects of a cost-reducing R&D investment by an entrant in a monopolistic market. In our analysis, we demonstrate a new aspect of "commitment effect" induced by such an irreversible investment. Given prior literature, it may appear intuitive that, if the entrant's R&D investment cannot result in a lower marginal cost than that of the incumbent, the entrant has no incentive to operate in the market. However, our results show that this prediction may be erroneous when an R&D investment by the entrant introduces technological (cost) uncertainty prior to the pricing stage.

In our application, we do not focus on the incumbent firm's R&D strategy. However, we believe that our basic analysis is potentially useful for future work in theoretical studies of sequential cost-reducing R&D or sequential entry with technological uncertainty.

Appendix A: Proof of Theorem 4.1

First, we establish some general properties of firms' equilibrium payoffs.

Lemma 6.1. *In any equilibrium,*

- $\Pi_U^* > 0$*;*
- *if there exists a* $k \in \{1, \cdots, K\}$ *such that* $c_k \leq c_U$*, then* $\Pi_k^* > 0$*;*
- $\Pi_K^* = 0$*.*

Proof. Firms can always avoid negative profit by setting prices above their marginal costs: $\Pi_l^* \geq 0$ for $l \in \{U, 1, \ldots, K\}$. Since the given strategy profile (P_U^*, P_I^*) is an equilibrium, the following must be satisfied:

$$\Pi_U^* \geq E[\pi_U(p_U, p_I)|p_U] \text{ for any } p_U \in \mathbb{R}_+.$$

where $E[\pi_U(p_U, p_I)|p_U] = \sum_{k=1}^K \eta_k \int \pi_U(p_U, p_I) P_k^*(dp_I)$.

Now suppose that $\Pi_U^* = 0$. We immediately have $E[\pi(p_U, p_I)|p_U] = 0$ for $p_U \in (c_U, c_K)$. This implies that $\overline{p}_k \leq c_U$ for all k. Hence, it must be satisfied that $\underline{p}_k \leq \overline{p}_k \leq \underline{p}_U$ for all k such that $c_k > c_U$. This inequality indicates that for all k such that $c_k > c_U$,

$$E[D_I(p_k, p_U)] \equiv \int D_I(p_k, p_U) P_U^*(dp_U) > 0 \ \text{ for } p_k \in \text{supp}(P_k^*).$$

Therefore, we must have $\Pi_k^* < 0$ for all k such that $c_k > c_U$. This outcome contradicts the fact that the given strategy profile is an equilibrium since there is a profitable deviation for firm I with the marginal cost of $c_k > c_U$. Thus, it must be true that $\Pi_U^* > 0$. Moreover, $\Pi_U^* > 0$ means that $\underline{p}_U > c_U$. Therefore, we immediately obtain the fact that $\Pi_k^* > 0$ for k such that $c_k \le c_U$.

Since $\Pi_U^* > 0$, we must have $\overline{p}_U \le \max_k \overline{p}_k$. Moreover, since firms share the demand equally in case of a tie, $\{p_U = \max_k \overline{p}_k\}$ has measure zero and $\{p_k \ge \overline{p}_U\}$ has a positive measure for some k on the probability space endowed with the equilibrium strategies P_U^* and P_I^*, respectively. Therefore, $\Pi_k^* = 0$ for some k.

Now, suppose that $\Pi_K^* > 0$. Then, Remark 4.1 shows that firm I of type $k < K$ can obtain a positive profit. This result contradicts the fact that $\Pi_k^* = 0$ for some k. Hence, we must have $\Pi_K^* = 0$. □

The following Proposition 6.1 shows the general properties of firms' equilibrium strategies.[23]

Proposition 6.1. *In any equilibrium,*

- P_U^* *is a cumulative distribution function on* $[\underline{p}_U, \overline{p}_U]$ *with a probability mass only at* $\overline{p}_U$*;*
- *for any* $k \le k_+^*$, P_k^* *is a cumulative distribution function, which does not have a probability mass on* $[\underline{p}_k, \overline{p}_k]$ *where* $\max\{c_k, c_U\} < \underline{p}_k < \overline{p}_k \le c_{k_0^*}$, $\overline{p}_{k-1} = \underline{p}_k$*, and* $[\underline{p}_U, \overline{p}_U] = [\underline{p}_1, \overline{p}_{k_+^*}]$*;*
- *for any* $k \ge k_0^*$, P_k^* *does not have a probability mass at* $\overline{p}_U$*, and* $\underline{p}_k \ge \overline{p}_U$*. Moreover,* $\underline{p}_{k'} = \overline{p}_U$ *for some* $k' \ge k_0^*$*.*

To prove this proposition, it will suffice to prove the following Lemmas 6.2–6.5.

The highest element in the support of all type $k \le k_+^*$ firm I's price distribution is denoted by $\overline{p}_+ \equiv \max_{k \le k_+^*} \overline{p}_k$. The lowest element in the support of all type $k \le k_+^*$ firm I's price distribution is denoted $\underline{p}_0 \equiv \min_{k \ge k_0^*} \underline{p}_k$. For $0 \le a < b$, let $\mathrm{P}_l^*(a, b)$ denote the probability with which firm $l \in \{U, 1, \ldots, K\}$ sets its price in $(a, b]$ under the given firms' strategies: $\mathrm{P}_l^*(a, b) \equiv P_l^*(b) - P_l^*(a)$. Let $\mathrm{P}_l^*(b, b)$ denote the probability mass at b: $\mathrm{P}_l^*(b, b) \equiv P_l^*(b) - \lim_{a \uparrow b} P_l^*(a)$.

Lemma 6.2. *In any equilibrium,*

- $c_{k_+^*} < \overline{p}_+ = \overline{p}_U = \underline{p}_0 \le c_{k_0^*}$*;*
- *Firm U sets* $\hat{p} \equiv \overline{p}_+ = \overline{p}_U = \underline{p}_0$ *with a positive probability, whereas any type of firm I never sets* $\hat{p}$ *with a positive probability.*

[23] As can be seen in Remark 4.2, the continuity of the action space makes the characterization of the equilibrium strategies difficult. The following fact is useful to discuss the optimality of the firms' strategies. We use this fact recurringly in the proof of Proposition 6.1. Fact: Fix all firms' strategies except that of firm l's for $l \in \{U, 1, \ldots, K\}$. If there exists an open subset $\tilde{A}$ of $\mathbb{R}_+$ such that no p_l in $\tilde{A}$ is optimal for firm l against the rival's strategy, then any of firm l's strategies which has a positive measure on $\tilde{A}$ is not a best response to the given strategy of its rival.

Proof. Fix an equilibrium. By Remark 4.1, we obtain $c_k < \underline{p}_k \leq \overline{p}_k \leq \overline{p}_U$ for any $k \leq k_+^*$. Moreover, since $\Pi_{k_0^*}^* = 0$, we must have $\overline{p}_U \leq c_{k_0^*}$. Therefore, we obtain $\overline{p}_U \leq \underline{p}_k$ for any $k \geq k_0^*$. Summarizing the above, we obtain $c_{k_+^*} < \overline{p}_+ \leq \overline{p}_U \leq \underline{p}_0$.

Now suppose that $\overline{p}_+ < \underline{p}_0$; that is, firm I never sets its price in $(\overline{p}_+, \underline{p}_0)$. Then, clearly, firm U never sets its price in this interval: $(\overline{p}_+, \underline{p}_0) \cap \mathrm{supp}(P_U^*) = \emptyset$. If $\lim_{\epsilon \downarrow 0} \mathrm{P}_U^*(\overline{p}_+ - \epsilon, \overline{p}_+) \equiv \mathrm{P}_U^*(\overline{p}_+, \overline{p}_+) = 0$, firm I of type $k \leq k_+^*$ has no incentive to set its price in $[\overline{p}_+ - \epsilon', \overline{p}_+]$ for small $\epsilon' > 0$.[24] Therefore, if we suppose that $\overline{p}_+ < \underline{p}_0$, it must be satisfied that firm U sets its price at $\overline{p}_+$ with a positive probability: $\mathrm{P}_U^*(\overline{p}_+, \overline{p}_+) > 0$. In this case, however, if $\mathrm{P}_k^*(\overline{p}_+, \overline{p}_+) = 0$ for any $k \leq k_+^*$, firm U has no incentive to set its price at $p_U = \overline{p}_+$ because setting its price slightly lower than $\underline{p}_0$ is more profitable. Moreover, if $\mathrm{P}_k^*(\overline{p}_+, \overline{p}_+) > 0$ for $k \leq k_+^*$, firm U can secure its demand by setting its price slightly lower than $\overline{p}_+$ since firms share the demand equally in case of a tie. Consequently, it is a necessary condition for the equilibrium to hold that $c_{k_+^*} < \overline{p}_+ = \overline{p}_U = \underline{p}_0$. Since firm I of type $k \geq k_0^*$ obtains zero profit, we deduce that $\underline{p}_0 \leq c_{k_0^*}$.

To ensure $\Pi_k^* > 0$ for all $k \leq k_+^*$, firm U must set $\hat{p} \equiv \overline{p}_+ = \overline{p}_U = \underline{p}_0$ with a positive probability.[25] Furthermore, because of the tie-breaking rule, no type of firm I sets $\hat{p}$ with a positive probability. This completes the proof of Lemma 6.2. □

Lemma 6.3. *In any equilibrium,*

- *if P_U^* has a probability mass at $\tilde{p} > c_{\tilde{k}}$ for $\tilde{k} \leq k_+^*$, then there exists an $\epsilon' > 0$ such that firm I of type $k \leq \tilde{k}$ never sets its price in $(\tilde{p}, \tilde{p} + \epsilon')$;*
- *similarly, if P_k^* a has probability mass at $\tilde{p} > c_U$, then there exists an $\epsilon'' > 0$ such that firm U never sets its price in $(\tilde{p}, \tilde{p} + \epsilon'')$.*

Proof. Suppose that $\mathrm{P}_U^*(\tilde{p}, \tilde{p}) > 0$ at $\tilde{p} > c_{\tilde{k}}$. Then the expected payoff firm I of type $k \leq \tilde{k}$ obtains by setting $\tilde{p} - \epsilon'$ is

$$\begin{aligned}\Pi_k(\tilde{p} - \epsilon') &= \int (\tilde{p} - \epsilon' - c_k) D_I(\tilde{p} - \epsilon', p_U) P_U^*(dp_U) \\ &= (\tilde{p} - \epsilon' - c_k) D(\tilde{p} - \epsilon')[1 - P_U^*(\tilde{p} - \epsilon') + \mathrm{P}_U^*(\tilde{p} - \epsilon', \tilde{p} - \epsilon')/2].\end{aligned}$$

[24]More precisely, if $\mathrm{P}_U^*(\overline{p}_+, \overline{p}_+) = 0$ and $\mathrm{P}_U^*(\underline{p}_0, \underline{p}_0) > 0$, there exists $\epsilon' > 0$ such that firm I of type $k \leq k_+^*$ can obtain a higher expected profit by setting its price at (slightly lower than) $\underline{p}_0$ than by setting its price in $[\overline{p}_+ - \epsilon', \overline{p}_+]$. On the other hand, if $\mathrm{P}_U^*(\overline{p}_+, \overline{p}_+) = 0$ and $\mathrm{P}_U^*(\underline{p}_0, \underline{p}_0) = 0$, there exists $\epsilon' > 0$ such that firm I of type $k \leq k_+^*$ never sets its price in $[\overline{p}_+ - \epsilon', \overline{p}_+]$ to ensure that it obtains a strictly positive profit.

[25]Given the definition of $\overline{p}_+$, for arbitrary small $x > 0$ the interval $(\overline{p}_+ - x, \overline{p}_+]$ has a positive measure under P_k^* for some $k \leq k_+^*$, i.e., $\mathrm{P}_k^*(\overline{p}_+ - x, \overline{p}_+) > 0$. Therefore, to ensure $\Pi_k^* > 0$ for all $k \leq k_+^*$, firm U must set $\hat{p} \equiv \overline{p}_+ = \overline{p}_U = \underline{p}_0$ with a positive probability: $\mathrm{P}_U^*(\hat{p}_+, \hat{p}_+) > 0$.

By setting $\tilde{p}+\epsilon'$, it obtains

$$\begin{aligned}\Pi_k(\tilde{p}+\epsilon') &= \int (\tilde{p}+\epsilon'-c_k) D_I(\tilde{p}+\epsilon', p_U) P_U^*(dp_U)\\ &= (\tilde{p}+\epsilon'-c_k) D(\tilde{p}+\epsilon')[1-P_U^*(\tilde{p}+\epsilon') + \mathrm{P}_U^*(\tilde{p}+\epsilon', \tilde{p}+\epsilon')/2].\end{aligned}$$

Note that for any $\delta > 0$, there exists an $\epsilon > 0$ such that for $\epsilon' < \epsilon$, $\mathrm{P}_U^*(\tilde{p}\pm\epsilon', \tilde{p}\pm\epsilon') < \delta$.[26] Since $\mathrm{P}_U^*(\tilde{p}, \tilde{p}) > 0$, we obtain

$$\lim_{\epsilon'\downarrow 0}[\Pi_k(\tilde{p}-\epsilon') - \Pi_k(\tilde{p}+\epsilon')] = \mathrm{P}_U^*(\tilde{p}, \tilde{p})[(\tilde{p}-c_k)D(\tilde{p})] > 0.$$

Moreover, we must have $\Pi_k^* \geq \lim_{\epsilon'\downarrow 0}\Pi_k(\tilde{p}-\epsilon')$. Then the above inequality implies that if P_U^* has probability mass at some $\tilde{p}$, then there exists an $\epsilon' > 0$ such that firm I of type $k \leq \tilde{k}$ never sets its price in $(\tilde{p}, \tilde{p}+\epsilon')$.

Next, suppose that P_k^* has probability mass at $\tilde{p} > c_U$.[27] Then, by setting $\tilde{p}-\epsilon''$ and $\tilde{p}+\epsilon''$, firm U obtains

$$\Pi_U(\tilde{p}-\epsilon'') = \sum_{\ell=1}^{K} \eta_\ell \int (\tilde{p}-\epsilon''-c_U) D_U(\tilde{p}-\epsilon'', p_\ell) P_\ell^*(dp_\ell),$$

and

$$\Pi_U(\tilde{p}+\epsilon'') = \sum_{\ell=1}^{K} \eta_\ell \int (\tilde{p}+\epsilon''-c_U) D_U(\tilde{p}+\epsilon'', p_\ell) P_\ell^*(dp_\ell).$$

Therefore, we obtain

$$\lim_{\epsilon''\downarrow 0}[\Pi_U(\tilde{p}-\epsilon'') - \Pi_U(\tilde{p}+\epsilon'')] = \eta_k \mathrm{P}_k^*(\tilde{p}, \tilde{p})[(\tilde{p}-c_k)D(\tilde{p})] > 0.$$

Since firm U's equilibrium profit must be no less than $\Pi_U(\tilde{p}-\epsilon'')$, it never sets its price at $p_U \in (\tilde{p}, \tilde{p}+\epsilon'')$. This completes the proof of Lemma 6.3. □

Lemma 6.4. *In any equilibrium,*

- $\mathcal{S} \equiv \mathrm{supp}(P_U^*) = \bigcup_{k\leq k_+^*} \mathrm{supp}(P_k^*)$ *and* $\mathcal{S}$ *is an interval;*
- P_U^* *does not have probability mass except at* $\overline{p}_U$*;*
- *for any* $k \leq k_+^*$, $\mathrm{supp}(P_k^*)$ *is an interval such that* $\underline{p}_k < \overline{p}_k$ *and* $\overline{p}_{k-1} = \underline{p}_k$*, and* P_k^* *does not have probability mass in its support.*

Proof. We prove Lemma 6.4 in four steps.

[26] If this fact does not hold, then the neighborhood of $\tilde{p}$ always includes countably infinite elements with a probability mass higher than δ. In this case, the measure of the entire sample space is greater than one, which contradicts the axiom of a probability measure.

[27] Without loss of generality, we can assume that only one type of firm I sets its price at $\tilde{p}$ with a positive probability.

Step 1: We first show that $\text{supp}(P^*_U) = \bigcup_{k\le k^*_+} \text{supp}(P^*_k)$. Suppose that there exists an $x \in \mathbb{R}_+$ such that $x \in \text{supp}(P^*_U)$ and $x \notin \bigcup_{k\le k^*_+} \text{supp}(P^*_k)$. This means that there exists an open neighborhood of x denoted by $\chi(x) \equiv (\underline{x}, \overline{x}) \subset \mathbb{R}_+$ such that $\underline{x} < x < \overline{x} < \overline{p}_+$ and $\chi(x)$ has a positive measure under P^*_U but has measure zero under P^*_k for any k. Clearly, firm U's expected profit is strictly increasing in $p_U \in \chi(x)$. This results implies that firm U has an incentive to deviate from the given strategy P^*_U. In the similar manner, we can conclude that there does not exist an x such that $x \notin \text{supp}(P^*_U)$ and $x \in \text{supp}(P^*_k)$ for some $k \le k^*_+$. Hence, it must be satisfied that $\text{supp}(P^*_U) = \bigcup_{k\le k^*_+} \text{supp}(P^*_k)$.

Step 2: Next, we show that $\mathcal{S} \equiv \text{supp}(P^*_U) = \bigcup_{k\le k^*_+} \text{supp}(P^*_k)$ is an interval. Suppose that $\mathcal{S}$ is not an interval. Then, there exists an open interval $\mathcal{Z} = (\underline{z}, \overline{z}) \subset \mathbb{R}_+$ such that $\mathcal{Z} \cap \mathcal{S} = \emptyset$, $\underline{p}_U < \underline{z} < \overline{z} < \overline{p}_U$, and $\underline{z} \in \mathcal{S}$.[28] Let k' denote a representative type of firm I such that $\underline{z} \in \text{supp}(P^*_{k'})$. If $\mathrm{P}^*_U(\underline{z}, \underline{z}) = 0$, then there exists an $\epsilon' > 0$ such that setting $p_{k'} = (\underline{z} + \overline{z})/2$ is more profitable for firm I of type k' than setting $p_{k'} \in (\underline{z} - \epsilon', \underline{z}]$.[29] Hence, it is necessary that $\mathrm{P}^*_U(\underline{z}, \underline{z}) > 0$. In this case, if $\mathrm{P}^*_k(\underline{z}, \underline{z}) = 0$ for any $k \le k^*_+$, setting $p_U = (\underline{z} + \overline{z})/2$ is more profitable for firm U than setting $p_U = \underline{z}$.[30] Otherwise, since firms share the demand equally in case of a tie, firm U can secure the its demand by setting its price slightly lower than $\underline{z}$. In summary, if $\mathcal{S}$ is not an interval, there always exists a profitable deviation for some firm. Hence, $\mathcal{S}$ must be an interval.

Step 3: By Lemma 6.2–6.3 and the result in Step 2, we immediately deduce that P^*_U does not have probability mass except at $\overline{p}_U$ and that for any $k \le k^*_+$, P^*_k does not have probability mass in its support.

Step 4: In this step, we show that if $2 \le k \le k^*_+$, then $\overline{p}_{k-1} = \underline{p}_k$.[31] The results obtained in Step 2-3 and in Lemma 6.2 show that P^*_U is a cumulative probability distribution with a probability mass only at $\overline{p}_U$ and that P^*_k has no probability mass for any $k \le k^*_+$.

Firm I of type k is indifferent among all $p_k \in \text{supp}(P^*_k)$ (almost everywhere), that is, $(p_k - c_k)D(p_k)(1 - P^*_U(p_k))$ is constant on $\text{supp}(P^*_k)$. Then, $(p_k - c_{\hat{k}})D(p_k)(1 - P^*_U(p_k))$ is strictly increasing in p_k on $\text{supp}(P^*_k)$ for any $\hat{k} > k$, and $(p_k - c_{\tilde{k}})D(p_k)(1 - P^*_U(p_k))$ is strictly decreasing in p_k on $\text{supp}(P^*_k)$ for any $\tilde{k} < k$. Hence, for any $k \le k^*_+$, we obtain $\overline{p}_{k-1} \le \underline{p}_k$. Since $\bigcup_{k\le k^*_+} \text{supp}(P^*_k)$ is an interval and P^*_k does not

[28] $\mathcal{Z}$ is well-defined since $\mathcal{S}$ is a closed subset of $\mathbb{R}_+$.

[29] The fact $\underline{z} \in \mathcal{S}$ implies that $\mathrm{P}^*_U(\underline{z} - \epsilon, \underline{z}) > 0$ and $\mathrm{P}^*_{k'}(\underline{z} - \epsilon, \underline{z}) > 0$ for $k' \le k^*_+$. Since $\mathrm{P}^*_U(\underline{z}, \underline{z}) = 0$, there exists $\tilde{\epsilon} > 0$ such that the expected profit of type k' firm I varies continuously with respect to $p_{k'}$ in $(\underline{z} - \tilde{\epsilon}, \underline{z}]$. Moreover, since firm U never sets its price in $(\underline{z}, \overline{z})$, the expected profit of type k' firm I is strictly increasing in $p_{k'} \in [\underline{z}, \overline{z})$. Obviously, setting $p_{k'} = (\underline{z} + \overline{z})/2$ is more profitable for firm I of type k' than setting $p_{k'} = \underline{z}$. The continuity of the expected profit with respect to $p_{k'}$ ensures that we can take $\epsilon' \in (0, \tilde{\epsilon}]$ such that firm I of type k' prefers $p_{k'} = (\underline{z} + \overline{z})/2$ to $p_{k'} \in (\underline{z} - \epsilon', \underline{z}]$. Since $\mathrm{P}^*_{k'}(\underline{z} - \epsilon, \underline{z}) > 0$, the given $P^*_{k'}$ is not the best response to P^*_U.

[30] This result contradicts $\mathrm{P}^*_U(\underline{z}, \underline{z}) > 0$.

[31] Since $\overline{p}_{k-1}$ is not defined for $k = 1$, we only have to make sure the case of $2 \le k \le k^*_+$.

have a probability mass, it must hold that $\underline{p}_{k-1} < \overline{p}_{k-1} = \underline{p}_k$. This completes the proof of Lemma 6.4. □

Lemma 6.5. *In any equilibrium,* $\max\{c_k, c_U\} < \underline{p}_k$ *for* $k \leq k_+^*$.

Proof. Since $\Pi_U^* > 0$ and $\Pi_k^* > 0$ for $k \leq k_+^*$, the proof is immediate. □

Summarizing Lemma 6.2–6.5, we obtain Proposition 6.1.

Proof of Theorem 4.1 In this section, we complete the proof of Theorem 4.1.

Refinement 1 requires that $\underline{p}_k \geq c_k$ for all $k \geq k_0^*$. Hence, Lemma 6.2 yields that $\overline{p}_U = \overline{p}_{k_+^*} = \underline{p}_{k_0^*} = c_{k_0^*} \equiv c_{k_+^*+1}$ under Refinement 1.

Firm U's indifference conditions at the boundaries, $\underline{p}_k$ and $\overline{p}_k$, immediately yield the following condition. For $k \in \{1, \ldots, k_+^*\}$, the boundaries must satisfy that

$$\overline{p}_{k_+^*} = c_{k_+^*+1} \text{ and } \left\{1 - \sum_{\ell=1}^{k} \eta_\ell\right\}(\overline{p}_k - c_U)D(\overline{p}_k) = (\underline{p}_1 - c_U)D(\underline{p}_1). \tag{9}$$

Proposition 6.1 and condition (9) proves the second statement of Theorem 4.1.◇

Since firm I of type $k \leq k_+^*$ obtains a positive expected profit, we must have $\underline{p}_k = \overline{p}_{k-1} > c_k$ for any $k \leq k_+^*$. If we suppose that for some $\tilde{k} \leq k_+^*$,

$$\begin{aligned}\overline{\Pi}_U(k_+^* + 1) &\equiv \left\{1 - \sum_{\ell=1}^{k_+^*} \eta_\ell\right\}(c_{k_+^*+1} - c_U)D(c_{k_+^*+1}) \\ &\leq \left\{1 - \sum_{\ell=1}^{\tilde{k}-1} \eta_\ell\right\}(c_{\tilde{k}} - c_U)D(c_{\tilde{k}}) \equiv \overline{\Pi}_U(\tilde{k}).\end{aligned}$$

then, condition (9) yields that

$$\overline{\Pi}_U(k_+^* + 1) = \left\{1 - \sum_{\ell=1}^{\tilde{k}-1} \eta_\ell\right\}(\overline{p}_{\tilde{k}-1} - c_U)D(\overline{p}_{\tilde{k}-1}) \leq \left\{1 - \sum_{\ell=1}^{\tilde{k}-1} \eta_\ell\right\}(c_{\tilde{k}} - c_U)D(c_{\tilde{k}}).$$

This inequality implies that $\overline{p}_{\tilde{k}-1} = \underline{p}_{\tilde{k}} \leq c_{\tilde{k}}$ which contradicts the fact that firm I of type $\tilde{k} \geq k_+^*$ earns a positive profit in equilibrium. Therefore, the following inequality must be satisfied in the equilibrium:

$$\overline{\Pi}_U(k_+^* + 1) > \max_{\tilde{k} \leq k_+^*} \overline{\Pi}_U(\tilde{k}). \tag{10}$$

Under Refinement 1, clearly, we must have $\overline{\Pi}_U(k_+^* + 1) \geq \overline{\Pi}_U(k)$ for all $k > k_+^* + 1$.[32]

[32]Otherwise, setting its price at $p_U = c_k - \epsilon$ is a profitable deviation for firm U.

From the above discussion, we can conclude that $k^*_+ + 1 = \min\{\arg\max_{\tilde{k}} \overline{\Pi}_U(\tilde{k})\}$, that is, $k^*_+ + 1 = \overline{k}$ is a necessary condition for the equilibrium to conform to Refinement 1. This result proves the first statement of Theorem 4.1.$\diamond$

Finally, we prove the third statement of Theorem 4.1. The indifferent conditions for firm $l \in \{U, 1, \dots, k^*_+\}$ derive the undominated equilibrium strategies. By setting $p_k \in [\underline{p}_k, \overline{p}_k]$, firm I of type $k \in \{1, \dots, k^*_+\}$ obtains

$$\begin{aligned}\left\{1 - P^*_U(p_k)\right\}(p_k - c_k)D(p_k) &= \left\{1 - P^*_U(\underline{p}_k)\right\}(\underline{p}_k - c_k)D(\underline{p}_k)\\ &= \left\{1 - P^*_U(\overline{p}_k)\right\}(\overline{p}_k - c_k)D(\overline{p}_k),\end{aligned}$$

where $P^*_U(\underline{p}_1) = 0$, $P^*_U(\overline{p}_{k^*_+}) = 1$, and $\overline{p}_k = \underline{p}_{k+1}$. Solving the above simultaneous equations, we get (3) as firm U's equilibrium strategy.

By setting $p_U \in [\underline{p}_k, \overline{p}_k]$, firm U obtains

$$\left[1 - \left\{\sum_{\ell=1}^{k-1} \eta_\ell + \eta_k P^*_k(p_U)\right\}\right](p_U - c_U)D(p_U) = \left[1 - \sum_{\ell=1}^{k^*_+} \eta_\ell\right](c_{k^*_+ +1} - c_U)D(c_{k^*_+ +1}).$$

We thus get (4) as firm I's equilibrium strategy. This completes the proof of Theorem 4.1. $\square$

Remark 6.1. *Suppose that $(c_1 - c_U)D(c_1) \geq \overline{\Pi}_U(k)$ for all $k > 1$. Recall that $c_U < c_1$ in this case. Then, there does not exist k^*_+ that satisfies the condition (10). Therefore, in the undominated equilibrium, firm U serves the entire market demand at a price c_1 and firm I earns zero profit. In the equilibrium, firm U sets its price at c_1, firm I with c_1 sets its price according to the uniform distribution on $[c_1, c_1 + \varepsilon]$, and firm I of type $k > 1$ sets its price at c_k.*

Appendix B: Second-order condition in subsection 5.1

Recall that

$$\Pi'(\eta) = (\underline{p}_U(\eta) - c_1)D(\underline{p}_U(\eta)) + \eta \underline{p}'_U(\eta)\left[D(\underline{p}_U(\eta)) + D'(\underline{p}_U(\eta))(\underline{p}_U(\eta) - c_1)\right] - \gamma C'(\eta),$$

where $\underline{p}'_U(\eta) = -(c_2 - c_U)D(c_2)/[D(\underline{p}_U(\eta)) + D'(\underline{p}_U(\eta))(\underline{p}_U(\eta) - c_U)]$. Since $C''(\eta) \geq 0$ and $(\underline{p}_U(\eta) - c_1)D(\underline{p}_U(\eta))$ is strictly decreasing in $\eta \in (0, 1)$, a sufficient condition for the second-order condition to be satisfied is given by

$$\begin{aligned}&\frac{d}{d\eta}\left[-\eta \frac{D(\underline{p}_U(\eta)) + D'(\underline{p}_U(\eta))(\underline{p}_U(\eta) - c_1)}{D(\underline{p}_U(\eta)) + D'(\underline{p}_U(\eta))(\underline{p}_U(\eta) - c_U)}\right]\\ &= -\frac{D(\underline{p}_U(\eta)) + D'(\underline{p}_U(\eta))(\underline{p}_U(\eta) - c_1)}{D(\underline{p}_U(\eta)) + D'(\underline{p}_U(\eta))(\underline{p}_U(\eta) - c_U)}\\ &\quad + \eta(c_2 - c_U)(c_U - c_1)D(c_2) \times \frac{D''(\underline{p}_U(\eta))D(\underline{p}_U(\eta)) - 2\{D'(\underline{p}_U(\eta))\}^2}{\{D(\underline{p}_U(\eta)) + D'(\underline{p}_U(\eta))(\underline{p}_U(\eta) - c_U)\}^3} \leq 0.\end{aligned}$$

Since $\underline{p}_U(\eta) \in [c_U, c_2]$ and $D(p)(p-c_i)$ is increasing in $p \in (0, c_2)$ for $i = 1$ and U, we obtain $D(\underline{p}_U(\eta))+D'(\underline{p}_U(\eta))(\underline{p}_U(\eta)-c_U) > 0$ and $D(\underline{p}_U(\eta))+D'(\underline{p}_U(\eta))(\underline{p}_U(\eta)-c_1) > 0$. Therefore, the condition $D''(p)D(p) - 2\{D'(p)\}^2 \leq 0$ for $p \in [c_U, c_2]$ is a sufficient condition for the second-order condition to be satisfied.

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